HPC - A Name KEY

Trimester Review Date__________

Perform the indicated operation. Find the domain using interval notation.

1) $g(x) = 2x + 5$
$f(x) = x^2 + 5x$
Find $(g \cdot f)(x) = (2x+5)(x^2+5x)$
Distribute!

$\boxed{= 2x^3 + 15x^2 + 25x}$

$\boxed{D: (-\infty, \infty)}$

2) $f(t) = -3t + 1$
$g(t) = 2t + 5$
Find $\dfrac{f(t)}{g(t)} = \boxed{\dfrac{-3t+1}{2t+5}}$

$2t + 5 \neq 0$

$t \neq -\frac{5}{2}$

$\boxed{D: \left(-\infty, -\frac{5}{2}\right) \cup \left(-\frac{5}{2}, \infty\right)}$

3) $g(x) = -2x^2 + 4$
$f(x) = 2x^2 + 2$
Find $\dfrac{g(x)}{f(x)} = \dfrac{-2x^2+4}{2x^2+2} = \dfrac{-\cancel{2}(x^2-4)}{\cancel{2}(x^2+1)}$

$\boxed{= -\dfrac{(x+2)(x-2)}{x^2+1}}$

$x^2 + 1 \neq 0$
No real sol.

$\boxed{D: (-\infty, \infty)}$

4) $g(x) = 2x + 3$
$h(x) = 3x^3 - x$
Find $\left(\dfrac{g}{h}\right)(x) = \dfrac{2x+3}{3x^3 - x} = \boxed{\dfrac{2x+3}{x(3x^2-1)}}$

$x \neq 0$

$3x^2 - 1 \neq 0$

$x \neq \pm\sqrt{\frac{1}{3}}$

$D: \left(-\infty, -\sqrt{\frac{1}{3}}\right) \cup \left(-\sqrt{\frac{1}{3}}, 0\right) \cup \left(0, \sqrt{\frac{1}{3}}\right) \cup \left(\sqrt{\frac{1}{3}}, \infty\right)$

5) $f(x) = x^3 - 1$
$g(x) = -3x$
Find $f(g(x))$ and factor completely!

$f(g(x)) = (g(x))^3 - 1 = (-3x)^3 - 1$

$\boxed{= -27x^3 - 1}$

$= -1(27x^3 + 1)$ $\boxed{= -1(3x+1)(9x^2 - 3x + 1)}$

$\boxed{D: (-\infty, \infty)}$

6) $f(x) = \sqrt{x-1}$ $D: [1, \infty)$
$g(x) = \sqrt{x+3}$ $D: [-3, \infty)$

Find $f(g(x)) = \sqrt{g(x) - 1}$

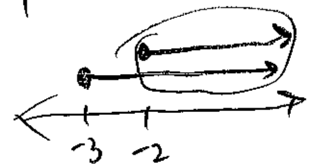
$= \sqrt{\sqrt{x+3} - 1}$

$\sqrt{x+3} - 1 \geq 0$

$(\sqrt{x+3})^2 \geq (1)^2$

$x + 3 \geq 1$
$-3 \quad -3$

$x \geq -2$



$\boxed{D_f: [-2, \infty)}$

7) $g(x) = x^2 - 2x$

$f(x) = \sqrt{3x+1}$ $D: [-\frac{1}{3}, \infty)$

Find $(g \circ f)(x) = g(f(x))$

$= (f(x))^2 - 2(f(x))$

$= (\sqrt{3x+1})^2 - 2(\sqrt{3x+1})$

$\boxed{= 3x+1 - 2\sqrt{3x+1}}$

$\boxed{D_F: [\frac{-1}{3}, \infty)}$

8) $h(x) = \sqrt{3x+4}$

$g(x) = -3x^2$

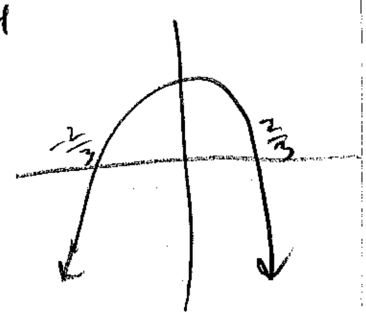
Find $h(g(x)) = \sqrt{3g(x)+4}$

$= \sqrt{3(-3x^2)+4}$

$= \sqrt{-9x^2+4}$

$= \sqrt{-1(9x^2-4)}$

$\boxed{= \sqrt{-1(3x-2)(3x+2)}}$



$-1(3x-2)(3x+2) \geq 0$

$\boxed{D: [-\frac{2}{3}, \frac{2}{3}]}$

Find the inverse of each function. Use proper notation!

9) $f(x) = \sqrt[3]{-\frac{x}{2}}$

$x^3 = \left(\sqrt[3]{-\frac{y}{2}}\right)^3$

$-2x^3 = -\frac{y}{2} \cdot -2$

$\boxed{f^{-1}(x) = -2x^3}$

10) $g(x) = \frac{7x+6}{3}$

$3x = \frac{7y+6}{3} \cdot 3$

$3x = 7y + 6$
$-6 \quad\quad -6$

$\frac{3x-6}{7} = \frac{7y}{7}$

$\boxed{g^{-1}(x) = \frac{3x-6}{7}}$

11) $f(x) = -(x-1)^3$

$\frac{x}{-1} = \frac{-(y-1)^3}{-1}$

$\sqrt[3]{-x} = \sqrt[3]{(y-1)^3}$

$\sqrt[3]{-x} = y - 1$
$+1 \quad\quad +1$

$\boxed{f^{-1}(x) = \sqrt[3]{-x} + 1}$

12) $h(x) = (x+1)^5 + 2$

$x = (y+1)^5 + 2$
$-2 \quad\quad -2$

$\sqrt[5]{x-2} = \sqrt[5]{(y+1)^5}$

$\sqrt[5]{x-2} = y + 1$
$-1 \quad\quad -1$

$\boxed{h^{-1}(x) = \sqrt[5]{x-2} - 1}$

Solve each equation. Show all work! Use any method here!

13) $8x^2 - 16x - 15 = 0$

$x = \frac{16 \pm \sqrt{(-16)^2 - 4(8)(-15)}}{2(8)}$

$x = \frac{16 \pm \sqrt{736}}{16} = \frac{16 \pm \sqrt{16 \cdot 46}}{16}$

$x = \frac{-16 \pm 4\sqrt{46}}{16} = \boxed{\frac{-4 \pm \sqrt{46}}{4}}$

14) $15x^2 - 8x - 17 = -8$

$+8 \quad +8$

$15x^2 - 8x - 9 = 0$

$x = \frac{8 \pm \sqrt{(-8)^2 - 4(15)(-9)}}{2(15)}$

$x = \frac{8 \pm \sqrt{604}}{30} = \frac{8 \pm \sqrt{4 \cdot 151}}{30}$

$x = \frac{8 \pm 2\sqrt{151}}{30} = \boxed{\frac{4 \pm \sqrt{151}}{15}}$

15) $7x^2 - 14x - 19 = 2$

$7x^2 - 14x - 21 = 0$

$7(x^2 - 2x - 3) = 0$

$7(x+1)(x-3) = 0$

$\boxed{x = -1, \ x = 3}$

Solve each equation by factoring.

16) $-3 + 14x = -5x^2$

$5x^2 + 14x - 3 = 0$

$(5x - 1)(x + 3) = 0$

$\boxed{x = \frac{1}{5}, \ x = -3}$

17) $4x^2 + 16x - 60 = -8x^2$

$12x^2 + 16x - 60 = 0$

$4(3x^2 + 4x - 15) = 0$

$4(3x - 5)(x + 3) = 0$

$\boxed{x = \frac{5}{3}, \ x = -3}$

18) $28x^2 + 61x + 36 = 3x^2 + x$

$25x^2 + 60x + 36 = 0$

$(5x+6)(5x+6) = 0$

$\boxed{x = -\frac{6}{5}}$

19) $27x^2 = -x^2 - 120 - 188x$

$28x^2 + 188x + 120 = 0$

$4(7x^2 + 47x + 30) = 0$

$4(7x+5)(x+6) = 0$

$\boxed{x = -\frac{5}{7},\ x = -6}$

Sketch a graph. Factor completely to find all complex solutions.

20) $f(x) = -125x^4 - 27x$

$= -x(125x^3 + 27)$

$(5x)^3 + (3)^3$

$= -x(5x+3)(25x^2 - 15x + 9)$

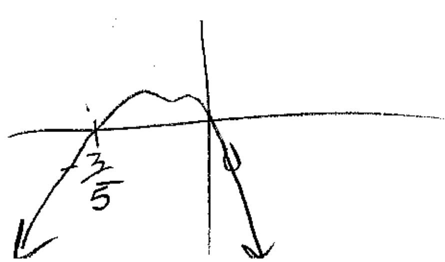
Real $x = 0,\ x = -\frac{3}{5}$

Complex $x = \frac{15 \pm \sqrt{(-15)^2 - 4(25)(9)}}{2(25)}$

$x = \frac{15 \pm \sqrt{-675}}{50}$

$x = \frac{15 \pm i\sqrt{225 \cdot 3}}{50}$

$x = \frac{15 \pm 15i\sqrt{3}}{50} = \boxed{\frac{3 \pm 3i\sqrt{3}}{10}}$



21) $f(x) = 128x^8 - 2x^2$

$= 2x^2(64x^6 - 1)$

$(4x^2)^3 - (1)^3$

$= 2x^2(4x^2 - 1)(16x^4 + 4x^2 + 1)$

$= 2x^2(2x+1)(2x-1)(16x^4 + 4x^2 + 1)$

Real $x = 0,\ x = -\frac{1}{2},\ x = \frac{1}{2}$

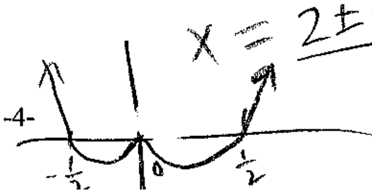
Because this part $64x^6 - 1$ was also the diff. of perfect squares, the quantity $(16x^4 + 4x^2 + 1)$ does split into 2 non factorable quantities. This type won't show up on the exam, but take a look.

$(16x^4 + 4x^2 + 1) = (4x^2 - 2x + 1)(4x^2 + 2x + 1)$

Complex: $x = \frac{2 \pm \sqrt{(-2)^2 - 4(4)(1)}}{2(4)}$

$x = \frac{2 \pm 2i\sqrt{3}}{8}$

$\boxed{x = \frac{\pm 1 \pm i\sqrt{3}}{4}}$



22) $f(x)=27x^3+64$

$(3x)^3+(4)^3$

$= (3x+4)(9x^2+12x+16)$

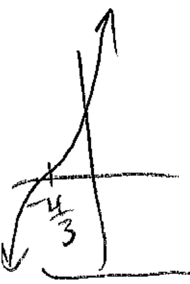
real $x=-\frac{4}{3}$

Complex: $x=\frac{-12\pm\sqrt{12^2-4(9)(16)}}{2(9)}$

$x=\frac{-12\pm\sqrt{-432}}{18}$

$x=\frac{-12\pm i\sqrt{144\cdot 3}}{18}$

$x=\frac{-12\pm 12i\sqrt{3}}{18}=\boxed{\frac{-2\pm 2i\sqrt{3}}{3}}$



$x(x^3-8x^2+21x-18)$

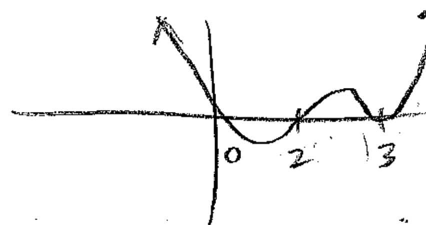
23) $f(x)=x^4-8x^3+21x^2-18x;\ f(2)=0$

$$
\begin{array}{r|l}
 & x^2-6x+9 \\
x-2 & x^3-8x^2+21x-18 \\
 & -(x^3-2x^2) \\
 & \quad -6x^2+21x \\
 & \quad -(-6x^2+12x) \\
 & \qquad 9x-18 \\
 & \qquad -(9x-18) \\
 & \qquad\quad 0
\end{array}
$$

$= x(x-2)(x^2-6x+9)$

$= x(x-2)(x-3)^2$

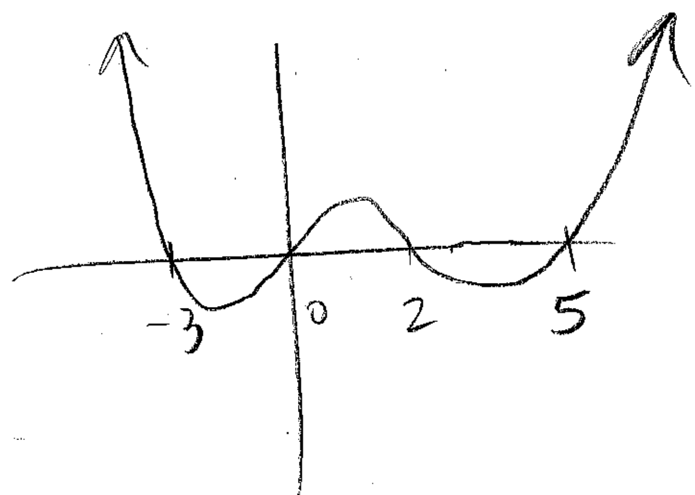
real: $x=0,\ x=2,\ x=3$ (bounce)



24) $f(x)=x^4-4x^3-11x^2+30x;\ f(5)=0$

$= x(x^3-4x^2-11x+30)$

$$
\begin{array}{r|l}
 & x^2+x-6 \\
x-5 & x^3-4x^2-11x+30 \\
 & -(x^3-5x^2) \\
 & \quad x^2-11x \\
 & \quad -(x^2-5x) \\
 & \qquad -6x+30 \\
 & \qquad -(-6x+30) \\
 & \qquad\quad 0
\end{array}
$$



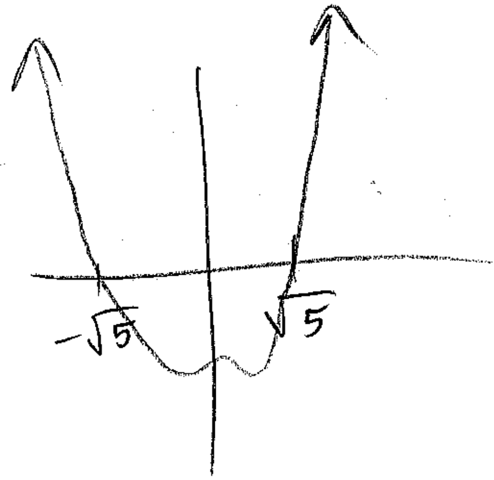
$= x(x-5)(x^2+x-6)$

$= x(x-5)(x+3)(x-2)$

$x=0,\ x=5,\ x=-3,\ x=2$

25) $f(x) = 25x^4 + 10x^3 - 124x^2 - 50x - 5;\; f\left(-\frac{1}{5}\right) = 0$

$$\begin{array}{r} 5x^3 + x^2 - 25x - 5 \\ 5x+1 \overline{)\, 25x^4 + 10x^3 - 124x^2 - 50x - 5} \\ -(25x^4 + 5x^3) \\ \hline 5x^3 - 124x^2 \\ -(5x^3 + x^2) \\ \hline -125x^2 - 50x \\ -(-125x^2 - 25x) \\ \hline -25x - 5 \\ -(-25x - 5) \\ \hline 0 \end{array}$$



$x^2(5x+1) - 5(5x+1)$

$= (5x+1)(5x^3 + x^2 - 25x - 5)$

$= (5x+1)(5x+1)(x^2 - 5)$

$= (5x+1)^2(x^2 - 5)$

real: $\boxed{x = \pm\sqrt{5}}$ $\boxed{\text{complex: } x = \pm i\sqrt{\tfrac{1}{5}}}$

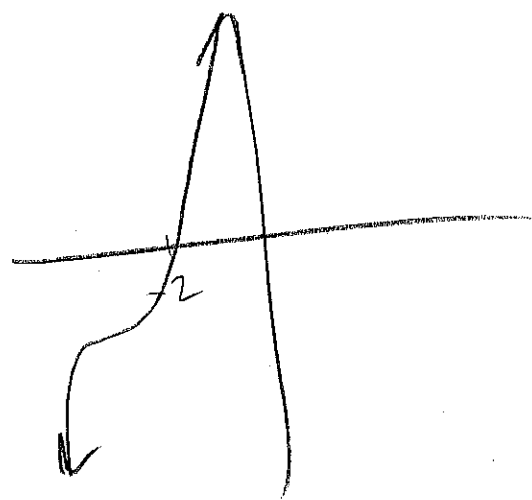
26) $f(x) = 2x^5 + 14x^3 + 16x^2 + 112$

$= 2(x^5 + 7x^3 + 8x^2 + 56)$

$x^3(x^2+7) + 8(x^2+7)$

$= 2(x^2+7)(x^3+8)$

$\boxed{= 2(x^2+7)(x+2)(x^2 - 2x + 4)}$



al $x = -2$

mplex: $\boxed{x = \pm i\sqrt{7}}$

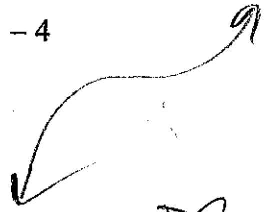
$x = \dfrac{2 \pm \sqrt{(-2)^2 - 4(1)(4)}}{2(1)}$

$x = \dfrac{2 \pm 2i\sqrt{3}}{2}$ $\boxed{x = 1 \pm 1i\sqrt{3}}$

Describe the end behavior of each function. Sketch the general shape if that is helpful.

27) $f(x) = x^3 - 2x^2 - 4$

Deg. 3

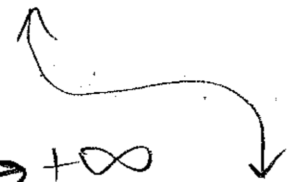


$x \to -\infty$, $y \to \underline{-\infty}$

$x \to +\infty$, $y \to \underline{+\infty}$

28) $f(x) = -x^5 + 4x^3 - 4x$

Deg 5



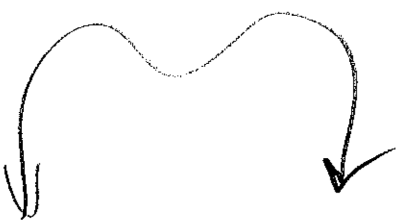
$x \to -\infty$, $y \to \underline{+\infty}$

$x \to +\infty$, $y \to \underline{-\infty}$

Sketch the general shape of each function. Nothing fancy, only looking for a very rough sketch!

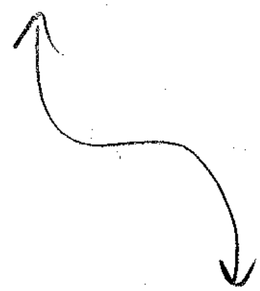
29) $f(x) = -x^4 + x^2 + x + 3$

Deg 4(-a)



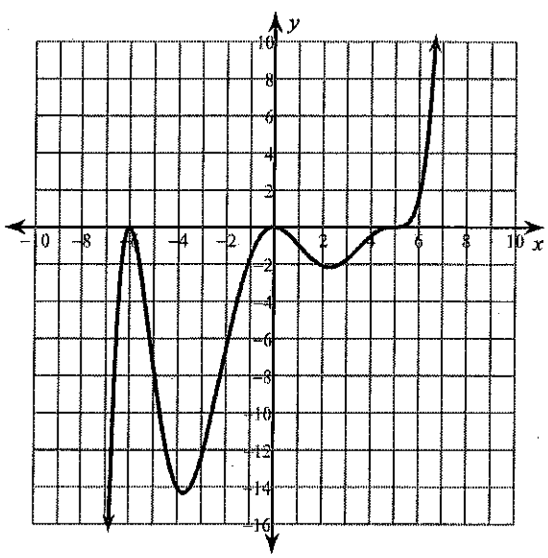
30) $f(x) = -x^5 + 3x^3 - 3x$

Deg 5(-a)



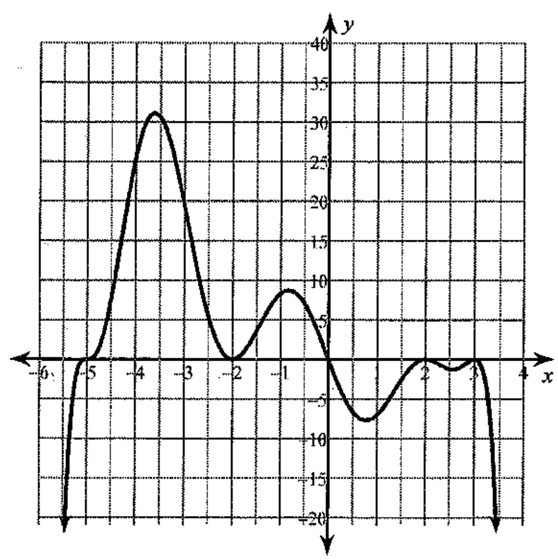
Write a function in factored form that would approximate the given graph.

31)



Equation: $x^2(x+6)^2(x-5)^3$

32)



Equation: $-x(x+5)^3(x+2)^2(x-2)^2(x-3)^2$

Identify the domain of each. Write your domain in interval notation. Domain only for these

33) $y = \sqrt{x+6}$

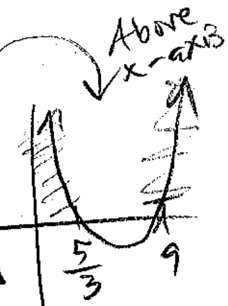
$D: [-6, \infty)$

$R: [0, \infty)$

34) $y = -\sqrt{(3x-5)(x-9)}$

$(3x-5)(x-9) \geq 0$

$\boxed{D: \left(-\infty, \frac{5}{3}\right] \cup [9, \infty)}$



Find the domain. Write your domain in interval notation. Factor and graph (only the radicand). One root has been given when needed.

35) $f(x) = \sqrt{x^5 - 3x^4 + 3x^3 - 3x^2 + 2x}$; $f(2) = 0$

$x(x^4 - 3x^3 + 3x^2 - 3x + 2)$

$$\begin{array}{r} x^3 - x^2 + x - 1 \\ x-2 \overline{\big)\, x^4 - 3x^3 + 3x^2 - 3x + 2} \\ -(x^4 - 2x^3) \\ \hline -x^3 + 3x^2 \\ -(-x^3 + 2x^2) \\ \hline x^2 - 3x \\ -(x^2 - 2x) \\ \hline -1x + 2 \\ (-1x + 2) \\ \hline 0 \end{array}$$

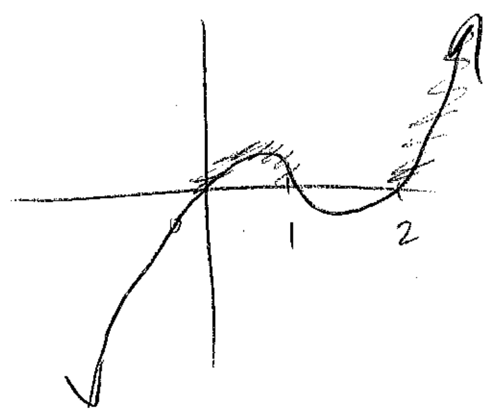
$x^2(x-1) + 1(x-1)$

$x(x-2)(x^3 - x^2 + x - 1) \geq 0$

$x(x-2)(x^2+1)(x-1) \geq 0$

No real sol.

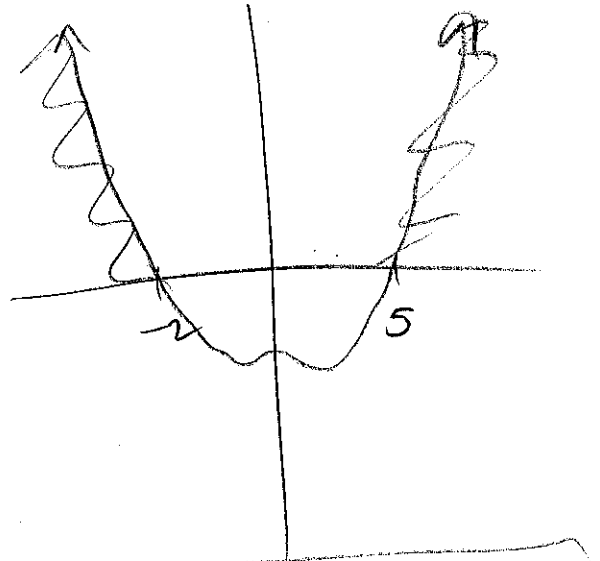
$x = 0,\ x = 2,\ x = 1$



$\boxed{D: [0, 1] \cup [2, \infty)}$

36) $f(x)=\sqrt{2x^4-6x^3-15x^2-15x-50};\ f(5)=0$

$$\begin{array}{r} 2x^3+4x^2+5x+10 \\ x-5\,\overline{\big)\,2x^4-6x^3-15x^2-15x-50} \\ -(2x^4-10x^3) \\ 4x^3-15x^2 \\ -(4x^3-20x^2) \\ 5x^2-15x \\ -(5x^2-25x) \\ 10x-50 \\ -(10x-50) \\ 0 \end{array}$$



$2x^2(x+2)+5(x+2)$

$(x-5)(2x^3+4x^2+5x+10)\geq 0$

$$\boxed{D:(-\infty,-2]\cup[5,\infty)}$$

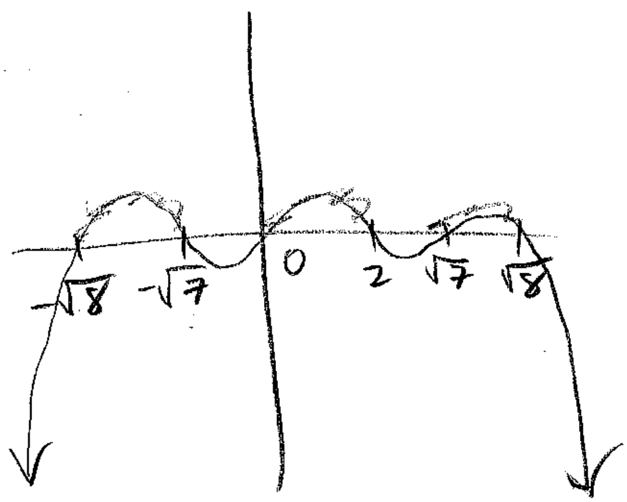
$(x-5)(2x^2+5)(x+2)\geq 0$

$x=5,\ x=-2$ → No real sol.

37) $f(x)=\sqrt{-2x^6+4x^5+30x^4-60x^3-112x^2+224x};\ f(2)=0$

$-2x(x^5-2x^4-15x^3+30x^2+56x-112)$

$$\begin{array}{r} x^4-15x^2+56 \\ x-2\,\overline{\big)\,x^5-2x^4-15x^3+30x^2+56x-112} \\ (x^5-2x^4) \\ 0 \\ -15x^3+30x^2 \\ -(-15x^3+30x^2) \\ +56x-112 \\ -(56x-112) \\ 0 \end{array}$$



$$\boxed{D:[-\sqrt{8},-\sqrt{7}]\cup[0,2]\cup[\sqrt{7},\sqrt{8}]}$$

$-2x(x-2)(x^4-15x^2+56)\geq 0$

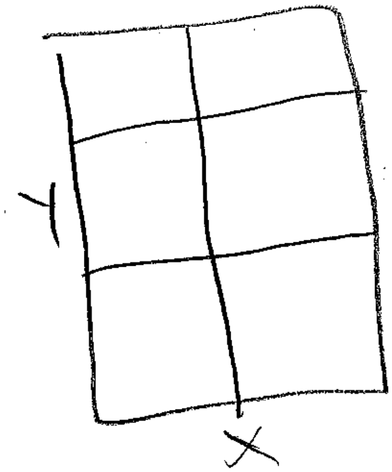
$-2x(x-2)(x^2-7)(x^2-8)\geq 0$

$x=0,\ x=2,\ x=\pm\sqrt{7},\ x=\pm\sqrt{8}$

$x=\pm 2\sqrt{2}$ ← equal/reduced

Maximum & Minimum situations.

38) Farmer Ted has 1000 ft. of chain-link fence to be used to construct six animal cages, as shown in the picture below.



a. Express the width y as a function of the length x.

$1000 = 3y + 4x$

$y = \frac{1000 - 4x}{3}$

b. Express the total enclosed area A of the cages as a function of x.

$A = xy \qquad A = x\left(\frac{1000}{3} - \frac{4}{3}x\right)$

c. Find the dimensions that maximize the enclosed area.

$A = -\frac{4}{3}x^2 + \frac{1000}{3}x$

$y = \frac{1000 - 4(125)}{3}$

x of vertex = $\frac{-\frac{1000}{3}}{2\left(-\frac{4}{3}\right)} = \boxed{125 \text{ ft}}$ $\boxed{y = 166.67 \text{ ft}}$

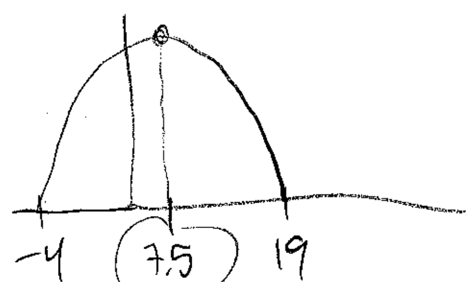
one dimension

39) Last year the yearbook at Central High cost \$95 and only 200 were sold. A student survey found that for every \$5 reduction in price, 50 more students will buy yearbooks. What price should be charged to maximize the revenue from yearbook sales?
(Hint: Total Revenue = (price of books)(number sold))

$\boxed{R = (95 - 5x)(200 + 50x) \qquad R = -250x^2 + 3750x + 19000}$

x-int: -4 & 19

represents # of reductions



$-\frac{b}{2a} = -\frac{3750}{2(-250)} = 7.5$ # of reductions to maximize revenue.

x of vertex using graphing strategy.

Price to Charge $= (95 - 5(7.5)) = \boxed{\$57.50}$

Solve each equation. Remember to check for extraneous solutions.

40) $\frac{6x-48}{x^2+2x}+\frac{1}{x^2+2x}=\frac{1(x+2)}{x(x+2)}$

$x(x+2)$ $x(x+2)$

$$\left[\frac{6x-47}{x(x+2)}=\frac{x+2}{x(x+2)}\right]x(x+2)$$

$$6x-47=x+2$$
$-x$ $+47$ $-x$ $+47$

$$\frac{5x}{5}=\frac{49}{5}$$

$$\boxed{x=\frac{49}{5}}$$

41) $\frac{6}{x^2+10x+16}-\frac{1}{x+2}\frac{(x+8)}{(x+8)}=\frac{1}{x^2+10x+16}$

$(x+2)(x+8)$ $(x+2)(x+8)$

$$(x+2)(x+8)\left[\frac{-x-2}{(x+2)(x+8)}=\frac{1}{(x+2)(x+8)}\right]$$

$$-x-2=1$$
$+2$ $+2$

$$\frac{-x}{-1}=\frac{3}{-1}$$

$$\boxed{x=-3}$$

Simplify.

42) $(5-2i)-(4+4i)$

$$\boxed{1-6i}$$

43) $(6+5i)-(3-3i)$

$$\boxed{3+8i}$$

44) $(-5+i)(-4+i)$

$$20-5i-4i+\underset{(-1)}{i^2}$$

$$\boxed{19-9i}$$

45) $(-3+7i)^2=9-21i-21i+49\underset{(-1)}{i^2}$

$$\boxed{-40-42i}$$

Rationalize & Simplify.

46) $\frac{(-9-7i)}{(-9+8i)}\cdot\frac{(-9-8i)}{(-9-8i)}=\frac{81+72i+63i+56\overset{(-1)}{i^2}}{81-64\underset{(-1)}{i^2}}$

$$=\frac{25+135i}{145}$$

$$\boxed{=\frac{5+27i}{29}}$$

47) $\frac{10i}{(6-7i)}\cdot\frac{(6+7i)}{(6+7i)}=\frac{60i+70\overset{(-1)}{i^2}}{36-49\underset{(-1)}{i^2}}$

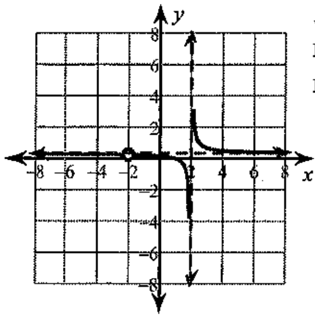
$$=\frac{-70+60i}{85}$$

$$\boxed{=\frac{-14+12i}{17}}$$

Identify the holes, vertical asymptotes, and horizontal asymptote of each. Then sketch the graph.

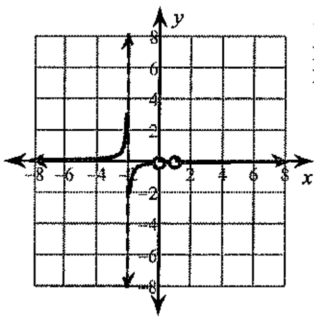
48) $f(x)=\frac{3}{x}$

A)



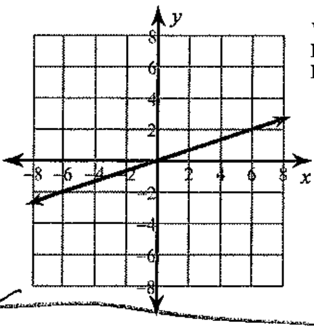
Vertical Asym.: $x=2$
Holes: $x=-2$
Horz. Asym.: $y=\frac{1}{3}$

B)



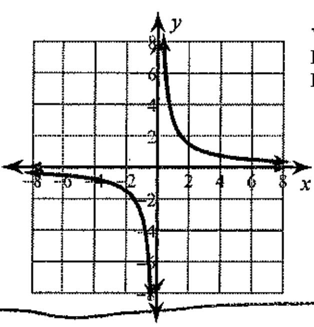
Vertical Asym.: $x=-2$
Holes: $x=0$, $x=1$
Horz. Asym.: $y=0$

C)



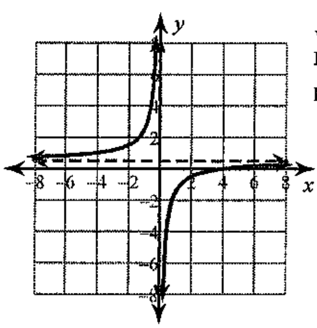
Vertical Asym.: None
Holes: None
Horz. Asym.: None

D)



Vertical Asym.: $x=0$
Holes: None
Horz. Asym.: $y=0$

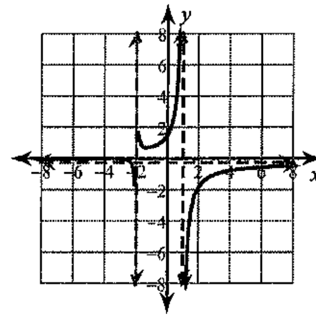
E)



Vertical Asym.: $x=0$
Holes: None
Horz. Asym.: $y=\frac{1}{2}$

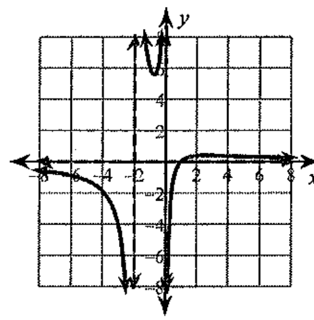
49) $f(x)=\frac{x^2+2x}{3x-3} = \frac{x(x+2)}{3(x-1)}$

A)



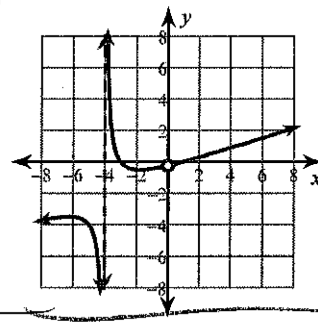
Vertical Asym.: $x=1$, $x=-2$
Holes: None
Horz. Asym.: $y=-\frac{1}{4}$

B)



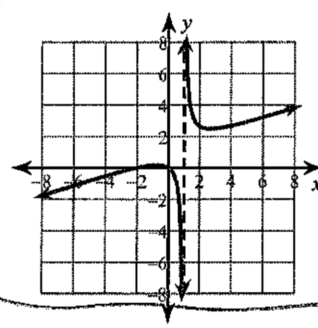
Vertical Asym.: $x=0$, $x=-2$
Holes: None
Horz. Asym.: $y=0$

C)



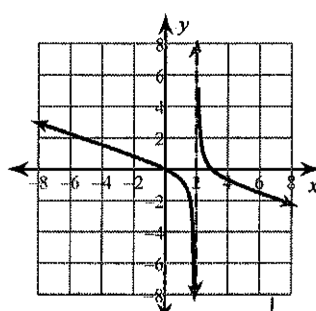
Vertical Asym.: $x=-4$
Holes: $x=0$
Horz. Asym.: None

D)



Vertical Asym.: $x=1$
Holes: None
Horz. Asym.: None

E)

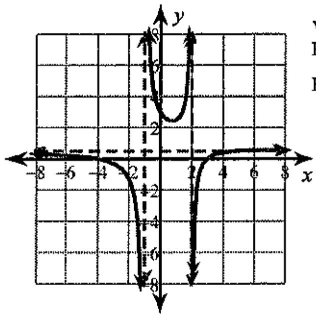


Vertical Asym.: $x=2$
Holes: None
Horz. Asym.: None

OOPS! This shouldn't be here! No slant asym on exam!

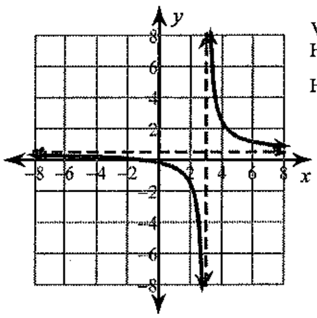
50) $f(x) = \dfrac{2x-6}{x+1} = \dfrac{2(x-3)}{x+1}$

A)



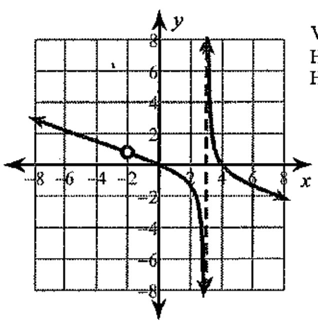
Vertical Asym.: $x = 2$, $x = -1$
Holes: None
Horz. Asym.: $y = \dfrac{1}{2}$

B)



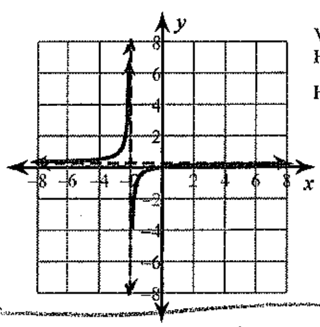
Vertical Asym.: $x = 3$
Holes: None
Horz. Asym.: $y = \dfrac{1}{2}$

C)



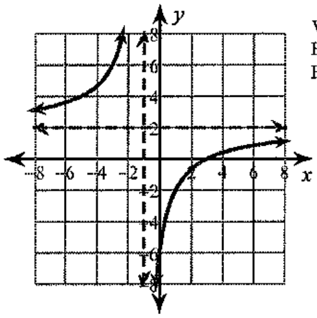
Vertical Asym.: $x = 3$
Holes: $x = -2$
Horz. Asym.: None

D)



Vertical Asym.: $x = -2$
Holes: None
Horz. Asym.: $y = \dfrac{1}{4}$

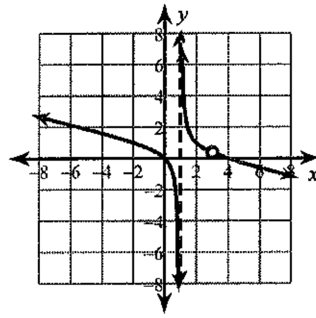
E)



Vertical Asym.: $x = -1$
Holes: None
Horz. Asym.: $y = 2$

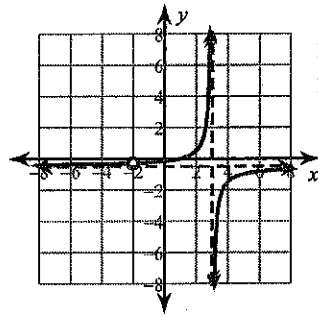
51) $f(x) = \dfrac{-2x-4}{x^2+5x+6} = \dfrac{-2\cancel{(x+2)}}{(x+3)\cancel{(x+2)}}$ Hole!

A)



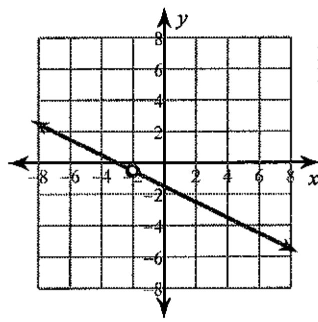
Vertical Asym.: $x = 1$
Holes: $x = 3$
Horz. Asym.: None

B)



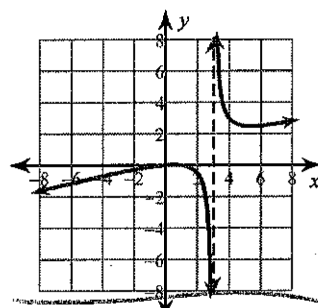
Vertical Asym.: $x = 3$
Holes: $x = -2$
Horz. Asym.: $y = -\dfrac{1}{2}$

C)



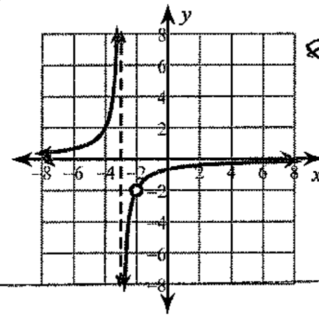
Vertical Asym.: None
Holes: $x = -2$
Horz. Asym.: None

D)



Vertical Asym.: $x = 3$
Holes: None
Horz. Asym.: None

E)



Vertical Asym.: $x = -3$
Holes: $x = -2$
Horz. Asym.: $y = 0$