# Here are the steps required for Adding and Subtracting Rational Expressions:

**Step 1**: Factor the denominator of each fraction to help find the LCD.

**Step 2**: Find the least common denominator (LCD).

**Step 3**: Find the new numerator for each fraction. To find the new numerators for each fraction compare the denominator of each of the original fractions to the LCD and write down everything different about the LCD in the numerator of that fraction. You should also consider using the letters "LCD" in the denominators instead of the actual LCD because this will make it less tempting to reduce the fractions.

**Step 4**: Combine the fraction by adding or subtracting the numerators and keeping the LCD. When subtracting, notice that the subtraction sign is moved into the numerator so it can be distributed later if needed.

**Step 5**: Simplify the numerator by distributing and combining like terms.

**Step 6**: Factor the numerator if you can and replace the letters "LCD" with the actual LCD.

**Step 7**: Simplify or reduce the rational expression if you can. Remember to reduce rational expressions the factors must be exactly the same in both the numerator and denominator.

**Example 1** – Simplify: $\dfrac{6}{x^2-4}+\dfrac{2}{x^2-5x+6}$

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| **Step 1**: Factor the denominator of each fraction to help find the LCD. | $\dfrac{6}{(x+2)(x-2)}+\dfrac{2}{(x-2)(x-3)}$ |
| **Step 2**: Find the least common denominator (LCD). | $LCD = (x+2)(x-2)(x-3)$ |
| **Step 3**: Find the new numerator for each fraction. If you compare the first fraction to the LCD the (x – 3) is different, so it was written in the numerator. In the second fraction the (x + 2) was different. | $\dfrac{6(x-3)}{LCD}+\dfrac{2(x+2)}{LCD}$ |
| **Step 4**: Combine the fraction by adding or subtracting the numerators and keeping the LCD. | $\dfrac{6(x-3)+2(x+2)}{LCD}$ |
| **Step 5**: Simplify the numerator by distributing and combining like terms. | $\dfrac{6x-18+2x+4}{LCD}=\dfrac{8x-14}{LCD}$ |
| **Step 6**: Factor the numerator if you can and replace the letters "LCD" with the actual LCD. | $\dfrac{2(4x-7)}{(x+2)(x-2)(x-3)}$ |
| **Step 7**: Simplify or reduce the rational expression if you can. In this case, the rational expression cannot be simplified. | $\dfrac{2(4x-7)}{(x+2)(x-2)(x-3)}$ |

**Example 2** – Simplify: $\dfrac{5}{2x^2+5x-3}-\dfrac{3}{2x^2-11x+5}$

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| **Step 1**: Factor the denominator of each fraction to help find the LCD. | $\dfrac{5}{(x+3)(2x-1)}-\dfrac{3}{(x-5)(2x-1)}$ |
| **Step 2**: Find the least common denominator (LCD). | $LCD=(x+3)(2x-1)(x-5)$ |
| **Step 3**: Find the new numerator for each fraction. If you compare the first fraction to the LCD the $(x-5)$ is different, so it was written in the numerator. In the second fraction the $(x+3)$ was different. | $\dfrac{5(x-5)}{LCD}-\dfrac{3(x+3)}{LCD}$ |
| **Step 4**: Combine the fraction by adding or subtracting the numerators and keeping the LCD. | $\dfrac{5(x-5)-3(x+3)}{LCD}$ |
| **Step 5**: Simplify the numerator by distributing and combining like terms. | $\dfrac{5x-25-3x-9}{LCD}=\dfrac{2x-34}{LCD}$ |
| **Step 6**: Factor the numerator if you can and replace the letters "LCD" with the actual LCD. | $\dfrac{2(x-17)}{(x+3)(2x-1)(x-5)}$ |
| **Step 7**: Simplify or reduce the rational expression if you can. In this case, the rational expression cannot be simplified. | $\dfrac{2(x-17)}{(x+3)(2x-1)(x-5)}$ |

# Here are the steps required for Multiplying Rational Expressions:

**Step 1**: Completely factor both the numerators and denominators of all fractions.

**Step 2**: Cancel or reduce the fractions. Remember that to reduce fractions; you can cancel something in the numerator with something in the denominator, but in order to cancel something in the numerator and denominator the two factors must be EXACTLY the same.

**Step 3**: Rewrite the remaining factor. Notice that you do NOT need to actually multiply anything in the numerator or denominator.

**Example 1** – Simplify: $\dfrac{x^2-3x-4}{x^2-x-2} \cdot \dfrac{x^2-4}{x^2+x-20}$

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| **Step 1**: Completely factor both the numerators and denominators of all fractions. | $\dfrac{(x-4)(x+1)}{(x+1)(x-2)} \cdot \dfrac{(x+2)(x-2)}{(x-4)(x+5)}$ |
| **Step 2**: Cancel or reduce the fractions. Remember that to reduce fractions; you can cancel something in the numerator with something in the denominator, but in order to cancel something in the numerator and denominator the two factors must be EXACTLY the same. | $\dfrac{\cancel{(x-4)}\cancel{(x+1)}}{\cancel{(x+1)}\cancel{(x-2)}} \cdot \dfrac{(x+2)\cancel{(x-2)}}{\cancel{(x-4)}(x+5)}$ |
| **Step 3**: Rewrite the remaining factor. Notice that you do NOT need to actually multiply anything in the numerator or denominator. | $\dfrac{x+2}{x+5}$ |

**Example 2** – Simplify: $\dfrac{2x^2+x-6}{3x^2-8x-3} \cdot \dfrac{x^2-7x+12}{2x^2-7x-4}$

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| **Step 1**: Completely factor both the numerators and denominators of all fractions. | $\dfrac{(2x-3)(x+2)}{(3x+1)(x-3)} \cdot \dfrac{(x-3)(x-4)}{(2x+1)(x-4)}$ |
| **Step 2**: Cancel or reduce the fractions. Remember that to reduce fractions; you can cancel something in the numerator with something in the denominator, but in order to cancel something in the numerator and denominator the two factors must be EXACTLY the same. | $\dfrac{(2x-3)(x+2)}{(3x+1)\cancel{(x-3)}} \cdot \dfrac{\cancel{(x-3)}\cancel{(x-4)}}{(2x+1)\cancel{(x-4)}}$ |
| **Step 3**: Rewrite the remaining factor. Notice that you do NOT need to actually multiply anything in the numerator or denominator. | $\dfrac{(2x-3)(x+2)}{(3x+1)(2x+1)}$ |

# Here are the steps required for Dividing Rational Expressions:

**Step 1**: Completely factor both the numerators and denominators of all fractions.

**Step 2**: Change the division sign to a multiplication sign and flip (or reciprocate) the fraction after the division sign; essential you need to multiply by the reciprocal.

**Step 3**: Cancel or reduce the fractions. Remember that to reduce fractions; you can cancel something in the numerator with something in the denominator, but in order to cancel something in the numerator and denominator the two factors must be EXACTLY the same.

**Step 4**: Rewrite the remaining factor. Notice that you do NOT need to actually multiply anything in the numerator or denominator.

**Example 1** – Simplify: $\dfrac{x^2 - 8x + 15}{x^2 - x - 6} \div \dfrac{x^2 + x - 20}{x^2 - 6x + 8}$

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| **Step 1**: Completely factor both the numerators and denominators of all fractions. | $\dfrac{(x-5)(x-3)}{(x-3)(x+2)} \div \dfrac{(x-4)(x+5)}{(x-2)(x-4)}$ |
| **Step 2**: Change the division sign to a multiplication sign and flip (or reciprocate) the fraction after the division sign; essential you need to multiply by the reciprocal. | $\dfrac{(x-5)(x-3)}{(x-3)(x+2)} \cdot \dfrac{(x-2)(x-4)}{(x-4)(x+5)}$ |
| **Step 3** : Cancel or reduce the fractions. Remember that to reduce fractions; you can cancel something in the numerator with something in the denominator, but in order to cancel something in the numerator and denominator the two factors must be EXACTLY the same. | $\dfrac{(x-5)\cancel{(x-3)}}{\cancel{(x-3)}(x+2)} \cdot \dfrac{(x-2)\cancel{(x-4)}}{\cancel{(x-4)}(x+5)}$ |
| **Step 4** : Rewrite the remaining factor. Notice that you do NOT need to actually multiply anything in the numerator or denominator. | $\dfrac{(x-5)(x-2)}{(x+2)(x+5)}$ |

**Example 2** – Simplify: $\dfrac{6x^2+11x+3}{2x^2-7x-15} \div \dfrac{9x^2-1}{2x^2-9x-5}$

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| **Step 1**: Completely factor both the numerators and denominators of all fractions. | $\dfrac{(3x+1)(2x+3)}{(2x+3)(x-5)} \div \dfrac{(3x+1)(3x-1)}{(x-5)(2x+1)}$ |
| **Step 2**: Change the division sign to a multiplication sign and flip (or reciprocate) the fraction after the division sign; essential you need to multiply by the reciprocal. | $\dfrac{(3x+1)(2x+3)}{(2x+3)(x-5)} \cdot \dfrac{(x-5)(2x+1)}{(3x+1)(3x-1)}$ |
| **Step 3** : Cancel or reduce the fractions. Remember that to reduce fractions; you can cancel something in the numerator with something in the denominator, but in order to cancel something in the numerator and denominator the two factors must be EXACTLY the same. | $\dfrac{\cancel{(3x+1)}\cancel{(2x+3)}}{\cancel{(2x+3)}\cancel{(x-5)}} \cdot \dfrac{\cancel{(x-5)}(2x+1)}{\cancel{(3x+1)}(3x-1)}$ |
| **Step 4** : Rewrite the remaining factor. Notice that you do NOT need to actually multiply anything in the numerator or denominator. | $\dfrac{2x+1}{3x-1}$ |