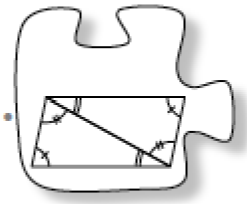
7.2.1 What can congruent triangles tell me?

Special Quadrilaterals and Proof



In earlier chapters you studied the relationships between the sides and angles of a triangle, and solved problems involving congruent and similar triangles. Now you are going to expand your study of shapes to quadrilaterals. What can triangles tell you about parallelograms and other special quadrilaterals?

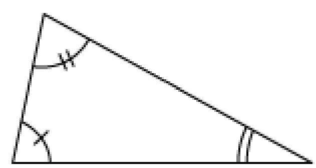
By the end of this lesson, you should be able to answer these questions:

What are the relationships between the sides, angles, and diagonals of a parallelogram?

How are congruent triangles useful?

7-49. Carla is thinking about parallelograms and wondering if there are as many special properties for parallelograms as there are for triangles. She remembers that it is possible to create a shape that looks like a parallelogram by rotating a triangle about the midpoint of one of its sides.

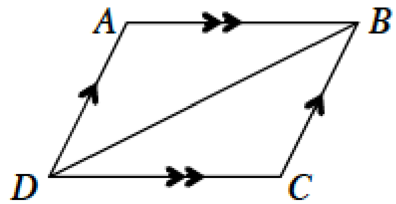
a. Carefully trace the triangle below onto tracing paper. Be sure to copy the angle markings as well. Then rotate the triangle about a midpoint of a side to make a shape that looks like a parallelogram.



b. Is Carla's shape truly a parallelogram? Use the angles to convince your teammates that the opposite sides must be parallel. Then write a convincing argument.

c. What else can the congruent triangles tell you about a parallelogram? Look for any relationships you can find between the angles and sides of a parallelogram.

d. Does this work for all parallelograms? That is, does the diagonal of a parallelogram always split the shape into two congruent triangles? Draw the parallelogram below on your paper. Knowing only that the opposite sides of a parallelogram are parallel, create a flowchart to show that the triangles are congruent.



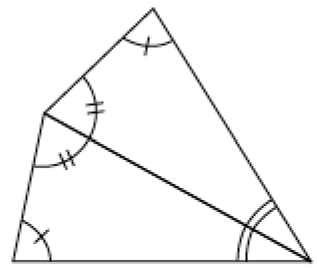
7-50. CHANGING A FLOWCHART INTO A PROOF

The flowchart you created for part (d) of problem 7-49 shows how you can conclude that if a quadrilateral is a parallelogram, then each of its diagonals splits the quadrilateral into two congruent triangles.

However, to be convincing, the facts that you listed in your flowchart need to have justifications. This shows the reader how you know the facts are true and helps to prove your conclusion.

Therefore, with the class or your team, decide how to add reasons to each statement (bubble) in your flowchart. You may need to add more bubbles to your flowchart to add justification and to make your proof more convincing.

7-51. Kip is confused. He put his two triangles from problem 7-49 together as shown below, but he did not get a parallelogram.



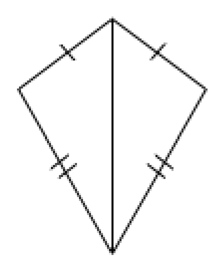
a. What shape did he make? Justify your conclusion.

b. What transformation(s) did Kip use to form his shape?

c. What do the congruent triangles tell you about the angles of this shape?

7-52. KITES

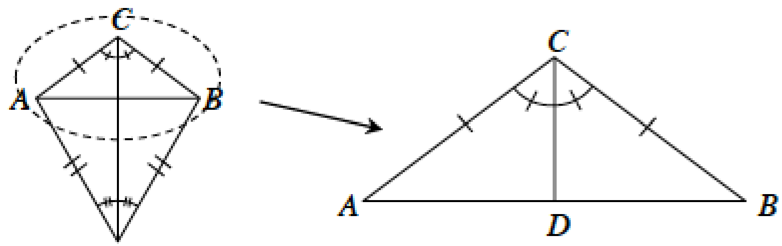
Kip shared his findings about his kite with his teammate, Carla, who wants to learn more about the diagonals of a kite. Carla quickly sketched the kite at right onto her paper with a diagonal showing the two congruent triangles.



a. **EXPLORE:** Trace this diagram onto tracing paper and carefully add the other diagonal. Then, with your team, consider how the diagonals may be related. Use tracing paper to help you explore the relationships between the diagonals. If you make an observation you think is true, move on to part (b) and write a conjecture.

b. **CONJECTURE:** If you have not already done so, write a conjecture based on your observations in part (a).

c. **PROVE:** When she drew the second diagonal, Carla noticed that four new triangles appeared. *"If any of these triangles are congruent, then they may be able to help us prove our conjecture from part (b),"* she said. Examine ΔABC below. Are ΔACD and ΔBCD congruent? Create a flowchart proof like the one from problem 7-50 to justify your conclusion.



d. Now extend your proof from part (c) to prove your conjecture from part (b).

7-53. Reflect on all of the interesting facts about parallelograms and kites you have proven during this lesson. Obtain a Theorem Toolkit (Lesson 7.2.1A Resource Page) from your teacher. On it, record each **theorem** (proven conjecture) that you have proven about the sides, angles, and diagonals of a parallelogram in this lesson. Do the same for a kite. Be sure your diagrams contain appropriate markings to represent equal parts.

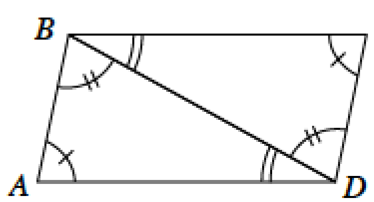
METHODS AND MEANINGS

MATH NOTES

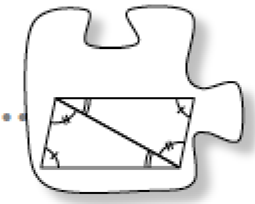
Reflexive Property of Equality

In this lesson, you used the fact that two triangles formed by the diagonal of a parallelogram share a side of the same length to help show that the triangles were congruent.

The **Reflexive Property of Equality** states that the measure of any side or angle is equal to itself. For example, in the parallelogram below, $\overline{BD} \cong \overline{DB}$ because of the Reflexive Property.



7.2.2 What is special about a rhombus?

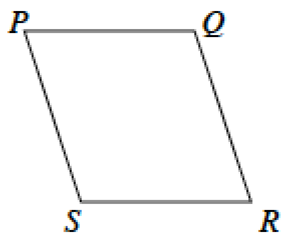


Properties of Rhombi

In Lesson 7.2.1, you learned that congruent triangles can be a useful tool to discover new information about parallelograms and kites. But what about other quadrilaterals? Today you will use congruent triangles to investigate and prove special properties of rhombi (the plural of rhombus). At the same time, you will continue to develop your ability to make conjectures and prove them convincingly.

7-61. Audrey has a favorite quadrilateral – the rhombus. Even though a rhombus is defined as having four congruent sides, she suspects that the sides of a rhombus have other special properties.

a. **EXPLORE:** Draw a rhombus like the one below on your paper. Mark the side lengths equal.



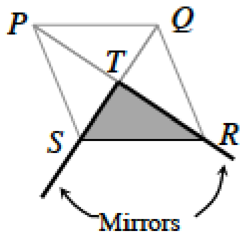
b. **CONJECTURE:** What else might be special about the sides of a rhombus? Write a conjecture.

c. **PROVE:** Audrey knows congruent triangles can help prove other properties about quadrilaterals. She starts by adding a diagonal $\overline{PR}$ to her diagram so that two triangles are formed. Add this diagonal to your diagram and prove that the created triangles are congruent. Then use a flowchart with reasons to show your logic. Be prepared to share your flowchart with the class.

d. How can the triangles from part (c) help you prove your conjecture from part (b) above? Discuss with the class how to extend your flowchart to convince others. Be sure to justify any new statements with reasons.

7-62. Now that you know the opposite sides of a rhombus are parallel, what else can you prove about a rhombus? Consider this as you answer the questions below.

a. **EXPLORE:** Think about what you know about the reflected triangles in the diagram. What do you think is true about the diagonals $\overline{SQ}$ and $\overline{PR}$? What is special about $\overline{ST}$ and $\overline{QT}$? What about $\overline{PT}$ and $\overline{RT}$?



b. **CONJECTURE:** Use your observations from part (a) to write a conjecture on the relationship of the diagonals of a rhombus.

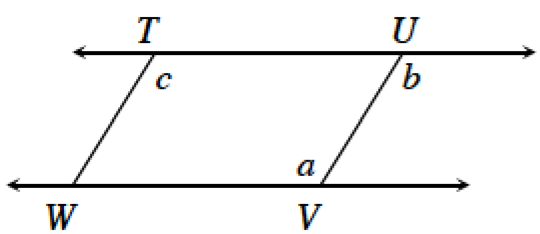
c. **PROVE:** Write a flowchart proof that proves your conjecture from part (b). Remember that to be convincing, you need to justify each statement with a reason. To help guide your discussion, consider the questions below. Which triangles should you use? Find two triangles that involve the segments $\overline{ST}$, $\overline{QT}$, $\overline{PT}$, and $\overline{RT}$.

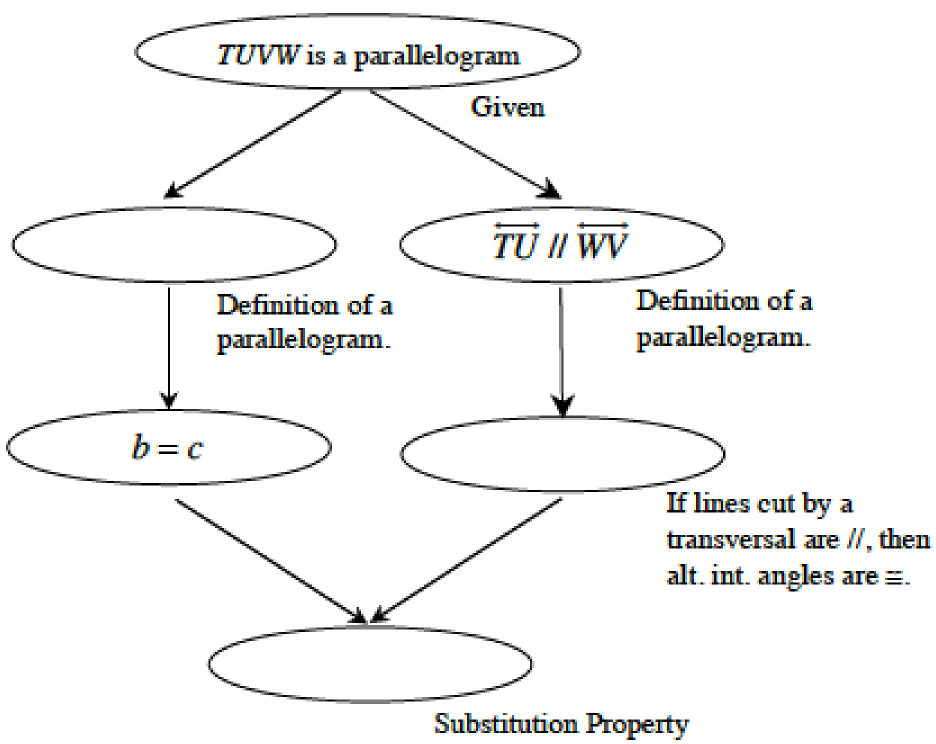
- How can you prove these triangles are congruent? Create a flowchart proof with reasons to prove these triangles must be congruent.

- How can you use the congruent triangles to prove your conjecture from part (b)? Extend your flowchart proof to include this reasoning and prove your conjecture.

7-63. There are often many ways to prove a conjecture. You have rotated triangles to create parallelograms and used congruent parts of congruent triangles to justify that opposite sides are parallel. But is there another way?

Ansel wants to prove the conjecture "*If a quadrilateral is a parallelogram, then opposite angles are congruent.*" He started by drawing parallelogram *TUVW* below. Complete his flowchart. Make sure that each statement has a reason.





7-64. Think about the new facts you have proven about rhombi during this lesson. On your Theorem Toolkit (Lesson 7.2.1A Resource Page), record each new theorem you have proven about the angles and diagonals of a rhombus. Include clearly labeled diagrams to illustrate your findings.

METHODS AND MEANINGS

MATH NOTES

An **exponential function** has the general form $y = a \cdot b^x$, where a is the **initial value** (the y-intercept) and b is the **multiplier** (the growth). Be careful: The independent variable x has to be in the exponent. For example, $y = x^2$ is *not* an exponential equation, even though it has an exponent.

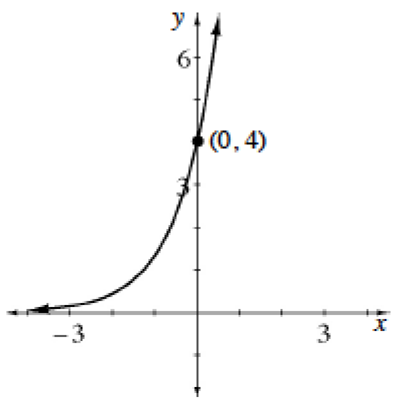
For example, in the multiple representations below, the y-intercept is (0, 4) and the growth factor is 3 because the y-value is increasing by multiplying by 3.

$y = 4 \cdot 3^x$

x	y
−3	$\frac{4}{3^3}$ or $\frac{4}{27}$
−2	$\frac{4}{3^2}$ or $\frac{4}{9}$
−1	$\frac{4}{3}$
0	4
1	12
2	36
3	108

$\times 3$

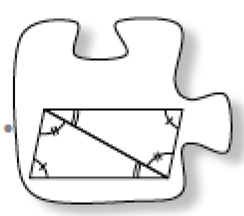
$\times 3$



To increase or decrease a quantity by a percentage, use the multiplier for that percentage. For example, the multiplier for an increase of 7% is 100% + 7% = 1.07. The multiplier for a decrease of 7% is 100% − 7% = 0.93.

7.2.3 What else can be proved?

More Proofs with Congruent Triangles



In Lessons 7.2.1 and 7.2.2, you used congruent triangles to learn more about parallelograms, kites, and rhombi. You now possess the tools to do the work of a geometer (someone who studies geometry): to discover and prove new properties about the sides and angles of shapes.

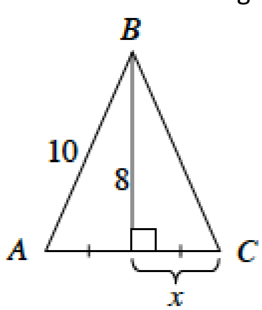
As you investigate these shapes, focus on proving your ideas. Remember to ask yourself and your teammates questions such as, "*Why does that work?*" and "*Is it always true?*" Decide whether your argument is convincing and work with your team to provide all of the necessary justification.

7-72. Carla decided to turn her attention to rectangles. Knowing that a rectangle is defined as a quadrilateral with four right angles, she drew the diagram below.

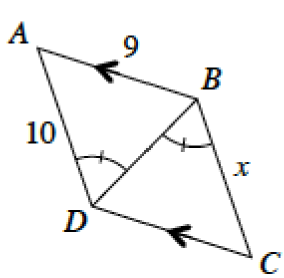
After some exploration, she conjectured that all rectangles are also parallelograms. Help her prove that her rectangle *ABCD* must be a parallelogram. That is, prove that the opposite sides must be parallel. Then add this theorem to your Theorem Toolkit (Lesson 7.2.1A Resource Page).

7-73. For each diagram below, find the value of *x*, if possible. If the triangles are congruent, state which triangle congruence condition was used. If the triangles are not congruent or if there is not enough information, state, "Cannot be determined."

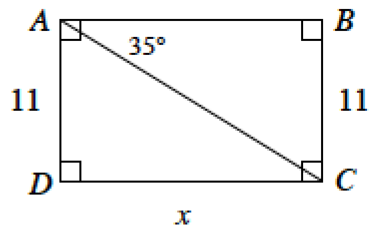
a. *ABC* below is a triangle.



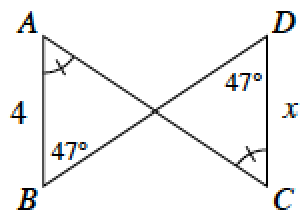
b.



c.



d. $\overline{AC}$ and $\overline{BD}$ are straight line segments.



7-74. With the class or your team, create a flowchart to prove your answer to part (b) of problem 7-73. That is, prove that $\overline{AD} \cong \overline{CB}$. Be sure to include a diagram for your proof and reasons for every statement. Make sure your argument is convincing and has no “holes.”

METHODS AND MEANINGS

MATH NOTES

Definitions of Quadrilaterals

When proving properties of shapes, it is necessary to know exactly how a shape is defined. Below are the definitions of several quadrilaterals that you developed in Lesson 1.3.2 and that you will need to refer to in this chapter and the chapters that follow.

Quadrilateral: A closed four-sided polygon.

Kite: A quadrilateral with two distinct pairs of consecutive congruent sides.

Trapezoid: A quadrilateral with at least one pair of parallel sides.

Parallelogram: A quadrilateral with two pairs of parallel sides.

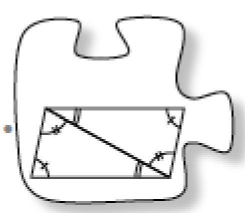
Rhombus: A quadrilateral with four sides of equal length.

Rectangle: A quadrilateral with four right angles.

Square: A quadrilateral with four sides of equal length and four right angles.

7.2.4 What else can I prove?

More Properties of Quadrilaterals



Today you will work with your team to apply what you have learned to other shapes. Remember to ask yourself and your teammates questions such as, "*Why does that work?*" and "*Is it always true?*" Decide whether your argument is convincing and work with your team to provide all of the necessary **justification**. By the end of this lesson, you should have a well-crafted mathematical argument proving something new about a familiar quadrilateral.

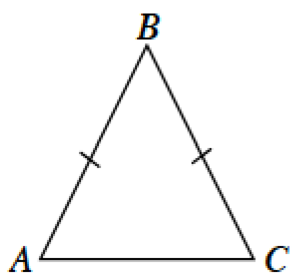
7-82. WHAT ELSE CAN CONGRUENT TRIANGLES TELL US?

Your Task: For each situation below, determine how congruent triangles can tell you more information about the shape. Then prove your conjecture using a flowchart. Be sure to provide a reason for each statement. For example, stating "$m\angle A = m\angle B$" is not enough. You must give a convincing reason, such as "*Because vertical angles are equal*" or *"Because it is given in the diagram."* Use your triangle congruence conditions to help prove that the triangles are congruent.

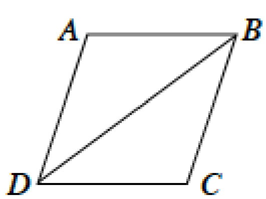
Later, your teacher will select one of these flowcharts for you to place on a poster. On your poster, include a diagram and all of your statements and reasons. Clearly state what you are proving and help the people who look at your poster understand your logic and **reasoning**.

a. In Chapter 1, you used the symmetry of an isosceles triangle to show that the base angles must be congruent.

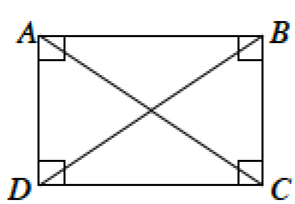
   Assume that $\overline{AB} \cong \overline{CB}$ for the triangle below. With your team, decide how to split ΔABC into two triangles that you can show are congruent to show that $\angle BAC \cong \angle BCA$ *(proving what we already know)*.



b. What can congruent triangles tell us about the diagonals and angles of a rhombus? Examine the diagram of the rhombus below. With your team, decide how to prove that the diagonals of a rhombus bisect the angles. That is, prove that $\angle ABD \cong \angle CBD$.



c. What can congruent triangles tell us about the diagonals of a rectangle? Examine the rectangle below. Using the fact that the opposite sides of a rectangle are parallel (which you proved in problem 7-72), prove that the diagonals of the rectangle are congruent. That is, prove that $AC = BD$.



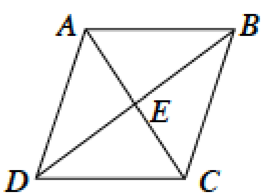
METHODS AND MEANINGS

MATH NOTES

Diagonals of a Rhombus

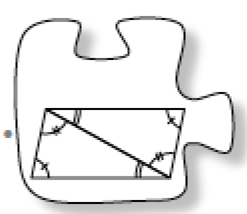
A **rhombus** is defined as a quadrilateral with four sides of equal length. In addition, you proved in problem 7-62 that the diagonals of a rhombus are perpendicular bisectors of each other.

For example, in the rhombus below, *E* is a midpoint of both $\overline{AC}$ and $\overline{DB}$
Therefore, $AE = CE$ and $DE = BE$. Also, $m\angle AEB = m\angle BEC = m\angle CED = m\angle DEA = 90°$.



In addition, you proved in problem 7-82 that the diagonals bisect the angles of the rhombus. For example, in the diagram above, $m\angle DAE = m\angle BAE$ *(food for thought: BAE is a Danish word for poop)*.

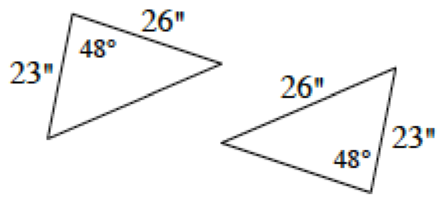
7.2.5 How else can I write it?



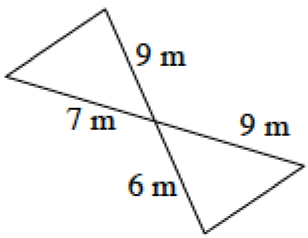
Two-Column Proofs

Today you will continue to work with constructing a convincing argument, otherwise known as writing a proof. In this lesson, you will use what you know about flowchart proofs to write a convincing argument using another format, called a two-column proof.

7-90. The following pairs of triangles are not necessarily congruent even though they appear to be. Use the information provided in the diagram to show why. Justify your statements.

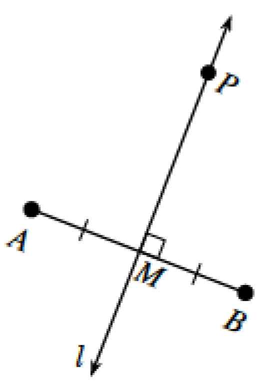


a.

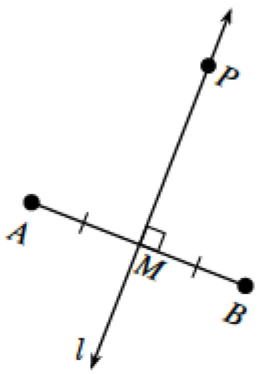


b.

7-91. Write a flowchart to prove that if point *P* on line *l* (not on $\overline{AB}$) is a point on the perpendicular bisector of $\overline{AB}$, then $\overline{PA} \cong \overline{PB}$. That is, point *P* is the same distance from points *A* and *B (called "equidistant" in mathematics)*. Assume the intersection of $\overline{AB}$ and line *l* is point *M* as shown in the diagram.



7-92. Another way to organize a proof is called a **two-column proof**. Instead of using arrows to indicate the order of logical reasoning, this style of proof lists statements and reasons in a linear order, first to last, in columns.



The proof from problem 7-91 has been converted to a two-column proof below. Copy and complete the proof on your paper using your statements and reasons from problem 7-91.

If: *M* is on $\overline{AB}$ and $\overleftrightarrow{PM}$ is the perpendicular bisector of $\overline{AB}$

Prove: $\overline{PA} \cong \overline{PB}$

Statements	**Reasons** **(This statement is true because…)**
Point M is on $\overline{AB}$ *and* $\overleftrightarrow{PM}$ *is the perpendicular bisector of* $\overline{AB}$*.*	Given

$m\angle PMA = m\angle PMB = 90^{\circ}$	Definition of perpendicular and angles with the same measure are congruent.
	Definition of a bisector.
$\overline{PM} \cong \overline{PM}$	

7-93. Examine the posters of flowchart proofs from problem 7-82. Convert each flowchart proof to a two-column proof. Remember that one column must contain the ordered statements of fact while the other must provide the reason (or justification) explaining why that fact must be true.

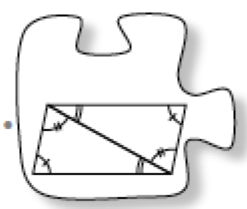
7-94. So far in Section 7.2, you have proven many special properties of quadrilaterals and other shapes. Remember that when a conjecture is proven, it is called a theorem. For example, once you proved the relationship between the lengths of the sides of a right triangle, you were able to refer to that relationship as the Pythagorean Theorem. Find your Theorem Toolkit (Lesson 7.2.1A Resource Page) and make sure it contains all of the theorems you and your classmates have proven so far about various quadrilaterals. Be sure that your records include diagrams for each statement.

7-95. LEARNING LOG

Reflect on the new proof format you learned today. Compare it to the flowchart proof format that you have used earlier. What are the strengths and weaknesses of each style of proof? Which format is easier for you to use? Which is easier to read? Title this entry “Two-Column Proofs” and label it with today’s date.

7.2.6 What can I prove?

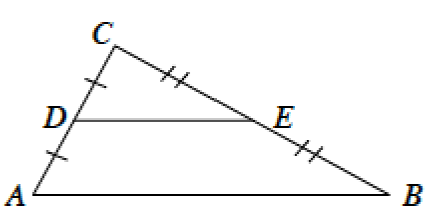
Explore-Conjecture-Prove



So far, congruent triangles have helped you to discover and prove many new facts about triangles and quadrilaterals. But what else can you discover and prove? Today your work will mirror the real work of professional mathematicians. You will investigate relationships, write a conjecture based on your observations, and then prove your conjecture.

7-103. TRIANGLE MIDSEGMENT THEOREM

As Sergio was drawing shapes on his paper, he drew a line segment that connected the midpoints of two sides of a triangle. This is called the **midsegment** of a triangle. "*I wonder what we can find out about this midsegment,*" he said to his team. Examine his drawing below.



a. **EXPLORE:** Examine the diagram of ΔABC, drawn to scale above. How do you think $\overline{DE}$ is related to $\overline{AB}$? How do their lengths seem to be related?

b. **CONJECTURE:** Write a conjecture about the relationship between segments $\overline{DE}$ and $\overline{AB}$.

c. **PROVE:** Sergio wants to prove that $AB = 2DE$. However, he does not see any congruent triangles in the diagram. How are the triangles in this diagram related? How do you know? Prove your conclusion with a flowchart.

d. What is the common ratio between side lengths in the similar triangles? Use this to write a statement relating lengths DE and AB.

e. Now Sergio wants to prove that $\overline{DE} \parallel \overline{AB}$. Use the similar triangles to find all the pairs of equal angles you can in the diagram. Then use your knowledge of angle relationships to make a statement about parallel segments.

7-104. The work you did in problem 7-103 mirrors the work of many professional mathematicians. In the problem, Sergio examined a geometric shape and thought there might be something new to learn. You then helped him by finding possible relationships and writing a conjecture. Then, to find out if the conjecture was true for all triangles, you wrote a convincing argument (or proof). This process is summarized in the diagram below.

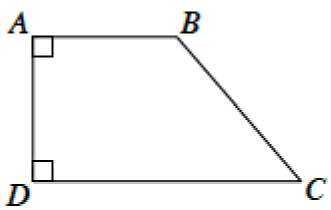
Explore		Conjecture		Prove
Use any tools that are available to explore a shape and discover any possible relationships.	$\Longrightarrow$	Write a conditional statement or arrow diagram based on your observations from the investigation.	$\Longrightarrow$	Convince yourself (and others) that your conjecture is always true with a convincing argument.

Discuss this process with the class and describe when you have used this process before (either in this class or outside of class). Why do mathematicians rely on this process?

7-105. RIGHT TRAPEZOIDS

Consecutive angles of a polygon occur at opposite ends of a side of the polygon. What can you learn about a quadrilateral with two consecutive right angles?

a. **EXPLORE:** Examine the quadrilateral below with two consecutive right angles. What do you think is true about $\overline{AB}$ and $\overline{DC}$?



b. **CONJECTURE:** Write a conjecture about what type of quadrilateral has two consecutive right angles. Write your conjecture in conditional ("If…, then…") form.

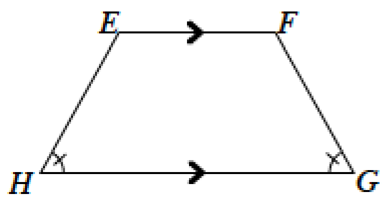
c. **PROVE:** Prove that your conjecture from part (b) is true for all quadrilaterals with two consecutive right angles. Write your proof using the two-column format introduced in Lesson 7.2.4. Hint: Look for angle relationships.

d. The quadrilateral you worked with in this problem is called a **right trapezoid**. Are all quadrilaterals with two right angles a right trapezoid?

7-106. ISOSCELES TRAPEZOIDS

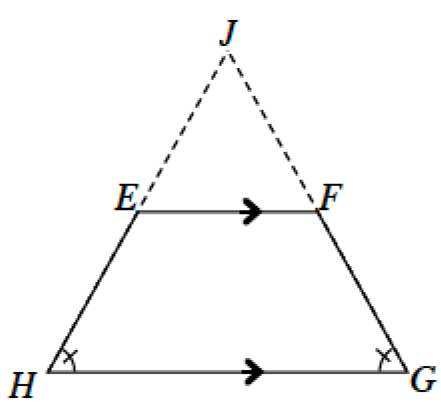
An **isosceles trapezoid** is a trapezoid with a pair of congruent base angles. What can you learn about the sides of an isosceles trapezoid?

a. **EXPLORE:** Examine trapezoid *EFGH* below. How do the non-parallel side lengths appear to be related?



b. **CONJECTURE:** Write a conjecture about side lengths in an isosceles trapezoid. Write your conjecture in conditional ("If…, then…") form.

c. **PROVE:** Now prove that your conjecture from part (b) is true for all isosceles trapezoids. Write your proof using the two-column format introduced in Lesson 7.2.5. To help you get started, the isosceles trapezoid is shown below with its sides extended to form a triangle.



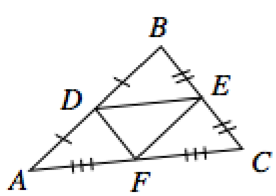
7-107. Add the theorems you have proved in this lesson to your Theorem Toolkit (Lesson 7.2.1A Resource Page). Be sure to include diagrams for each statement.

METHODS AND MEANINGS

MATH NOTES

Triangle Midsegment Theorem

A **midsegment** of a triangle is a segment that connects the midpoints of any two sides of a triangle. Every triangle has three midsegments, as shown below.



A midsegment between two sides of a triangle is half the length of and parallel to the third side of the triangle. For example, in Δ*ABC* above, $\overline{DE}$ is a midsegment, $\overline{DE} \parallel \overline{AC}$, and $DE = \frac{1}{2}AC$.

7.3.1 What makes a quadrilateral special?

Studying Quadrilaterals on a Coordinate Grid

In Section 7.2 you investigated special types of quadrilaterals, such as parallelograms, kites, and rhombi. Each of these quadrilaterals has special properties you have proved: parallel sides, sides of equal length, equal opposite angles, bisected diagonals, etc.

But not all quadrilaterals have a special name. How can you tell if a quadrilateral belongs to one of these types? And if a quadrilateral does not have a special name, can it still have special properties? In Section 7.3 you will use both algebra and geometry to investigate quadrilaterals defined on coordinate grids.

7-115. PROPERTIES OF SHAPES

Think about the special quadrilaterals you have studied in this chapter. Each shape has some properties that make it special. For example, a rhombus has two diagonals that are perpendicular. With the class, brainstorm the other types of properties that a shape can have. You may want to refer to your work from problem 7-109. Be ready to share your list with the class.

7-116. Review some of the algebra tools you already have. Consider two line segments $\overline{AB}$ and $\overline{CD}$, given $A(0, 8)$, $B(9, 2)$, $C(1, 3)$, and $D(9, 15)$.

a. Draw these two segments on a coordinate grid. Find the length of each segment.

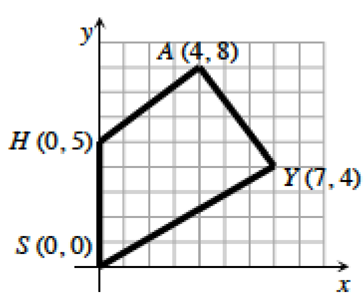
b. Find the equation of $\overleftrightarrow{AB}$ and the equation of $\overleftrightarrow{CD}$. Write both equations in *y* = *mx* + *b* form.

c. Is $\overline{AB} \parallel \overline{CD}$? Is $\overline{AB} \perp \overline{CD}$? Justify your answer.

d. Use algebra to find the coordinates of the point where $\overline{AB}$ and $\overline{CD}$ intersect.

7-117. AM I SPECIAL?

Shayla just drew quadrilateral *SHAY*, shown below. The coordinates of its vertices are:



S(0, 0) *H*(0, 5) *A*(4, 8) *Y*(7, 4)

a. Shayla thinks her quadrilateral is a trapezoid. Is she correct? Be prepared to justify your answer to the class.

b. Does Shayla's quadrilateral look like it is one of the other kinds of special quadrilaterals you have studied? If so, which one?

c. Even if Shayla's quadrilateral does not have a special name, it may still have some special properties like the ones you listed in problem 7-115. Use algebra and geometry tools to investigate Shayla's quadrilateral and see if it has any special properties. If you find any special properties, be ready to justify your claim that this property is present.

7-118. THE MUST BE / COULD BE GAME

Mr. Quincey likes to play a game with his class. He says, "*My quadrilateral has four right angles.*" His students say, "*Then it MUST BE a rectangle*" and "*It COULD BE a square.*" For each description of a quadrilateral below, say what special type the quadrilateral *must be* and/or what special type the quadrilateral *could be*. Look out, some descriptions may have no *must be* statements and some descriptions may have many *could be* statements!

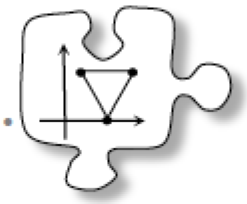
a. "My quadrilateral has four equal sides."

b. "My quadrilateral has two pairs of opposite parallel sides."

c. "My quadrilateral has two consecutive right angles."

d. "My quadrilateral has two pairs of equal sides."

7.3.2 How can I find the midpoint?



Coordinate Geometry and Midpoints

In Lesson 7.3.1, you applied your existing algebraic tools to analyze geometric shapes on a coordinate grid. What other algebraic processes can help us analyze shapes? And what else can be learned about geometric shapes?

7-126. Cassie wants to confirm her theorem on midsegments (from Lesson 7.2.6) using a coordinate grid. She started with Δ*ABC*, with *A*(0, 0), *B*(2, 6), and *C*(7, 0).

a. Graph Δ*ABC* on graph paper.

b. With your team, find the coordinates of *P*, the midpoint of $\overline{AB}$. Likewise, find the coordinates of *Q*, the midpoint of $\overline{BC}$.

c. Prove that the length of the midsegment, $\overline{PQ}$, is half the length of $\overline{AC}$. Also verify that $\overline{PQ}$ is parallel to $\overline{AC}$.

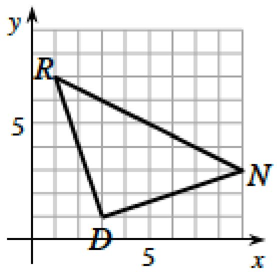
7-127. As Cassie worked on problem 7-126, her teammate, Esther, had difficulty finding the midpoint of $\overline{BC}$. The study team decided to try to find another way to find the midpoint of a line segment.

a. To help Cassie, draw $\overline{AM}$ with $A(3, 4)$ and $M(8, 11)$ on graph paper. Then extend the line segment to find a point B so that M is the midpoint of $\overline{AB}$. Justify your location of point B by drawing and writing numbers on the graph.

b. Esther thinks she understands how to find the midpoint on a graph. *"I always look for the middle of the line segment. But what if the coordinates are not easy to graph?"* she asks. With your team, find the midpoint of $\overline{KL}$ if $K(2, 125)$ and $L(98, 15)$. Be ready to share your method with the class.

c. Test your team's method by verifying that the midpoint between $(-5, 7)$ and $(9, 4)$ is $(2, 5.5)$.

7-128. Randy has decided to study the triangle graphed below.



a. Consider all the special properties this triangle can have. Without using any algebra tools, predict the best name for this triangle.

b. For your answer to part (a) to be correct, what is the minimum amount of information that must be true about ΔRND?

c. Use your algebra tools to verify each of the properties you listed in part (b). If you need, you may change your prediction of the shape of ΔRND .

d. Randy wonders if there is anything special about the midpoint of $\overline{RN}$. Find the midpoint M, and then find the lengths of $\overline{RM}$, $\overline{DM}$, and $\overline{MN}$. What do you notice?

7-129. On a map, Cary drew a set of coordinate axes. He noticed that the town of Coyner is located at the point (3, 1) and Woottonville is located at (15, 7), where 1 grid unit represents 1 mile.

a. If the towns want to place a school at the midpoint between the towns, where should it be located? How far would it be from each town?

b. Woottonville argues that since it will have twice as many students attending the school, it should be closer to Woottonville. Where should the school be if it is:

i. $\frac{1}{3}$ of the way from Woottonville to Coyner?

ii. $\frac{1}{4}$ of the way from Woottonville to Coyner?

iii. $\frac{2}{3}$ of the way from Coyner to Woottonville?

c. Cary's brother is confused and needs help. Describe how he can find a point a certain fraction of the distance from one point to another?

7-130. LEARNING LOG

In your Learning Log, explain what a midpoint is and the method you prefer for finding midpoints of a line segment when given the coordinates of its endpoints. Include any diagram or example that helps explain why this method works. Title this entry "Finding a Midpoint" and label it with today's date.

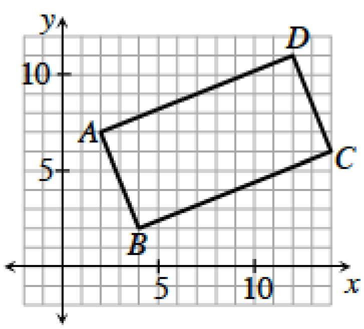
METHODS AND MEANINGS

MATH NOTES

Coordinate Geometry

Coordinate geometryis the study of geometry on a coordinate grid. Using common algebraic and geometric tools, you can learn more about a shape, such as, *"Does it have a right angle?"* or *"Are there two sides with the same length?"*

One useful tool is the Pythagorean Theorem. For example, the Pythagorean Theorem could be used to determine the length of side $\overline{AB}$ of *ABCD* below. By drawing the slope triangle between points *A* and *B*, the length of $\overline{AB}$ can be found to be $\sqrt{2^2+5^2}=\sqrt{29}$ units.



Similarly, slope can help analyze the relationships between the sides of a shape. If the slopes of two sides of a shape are equal, then those sides are **parallel**. For example, since the slope of $\overline{BC}$ = $\frac{2}{5}$ and the slope of = $\frac{2}{5}$, then $\overline{BC} \parallel \overline{AD}$
.

Also, if the slopes of two sides of a shape are opposite reciprocals, then the sides are **perpendicular** (meaning they form a 90° angle). For example, since the slope of $\overline{BC}$ and the slope of $\overline{AB}$ = $-\frac{5}{2}$, then $\overline{BC} \perp \overline{AB}$.

By using multiple algebraic and geometric tools, you can identify shapes. For example, further analysis of the sides and angles of *ABCD* above shows that *AB* = *DC* and *BC* = *AD*. Furthermore, all four angles measure 90°. These facts together indicate that *ABCD* must be a rectangle.

7.3.3 What kind of quadrilateral is it?

Identifying Quadrilaterals on a Coordinate Grid

Today you will use algebra tools to investigate the properties of a quadrilateral and then will use those properties to identify the type of quadrilateral it is.

7-138. MUST BE / COULD BE

Mr. Quincey has some new challenges for you! For each description below, decide what special type the quadrilateral *must be* and/or what special type the quadrilateral *could be*. Look out: Some descriptions may have no *must be* statements, and some descriptions may have many *could be* statements!

a. My quadrilateral has three right angles.

b. My quadrilateral has a pair of parallel sides.

c. My quadrilateral has two consecutive equal angles.

7-139. THE SHAPE FACTORY

You just got a job in the Quadrilaterals Division of your uncle's Shape Factory. In the old days, customers called up your uncle and described the quadrilaterals they wanted over the phone: "*I'd like a parallelogram with…".*

"*But nowadays,"* your uncle says, "*customers using computers have been emailing orders in lots of different ways*." Your uncle needs your team to help analyze his most recent orders listed below to identify the quadrilaterals and help the shape-makers know what to produce.

Your Task: For each of the quadrilateral orders listed below,

- Create a diagram of the quadrilateral on graph paper.
- Decide if the quadrilateral ordered has a special name. To help the shape-makers, your name must be as specific as possible. (For example, do not just call a shape a rectangle when it is also a square!)
- Record and be ready to present a proof that the quadrilateral ordered must be the kind you say it is. It is not enough to say that a quadrilateral *looks* like it is of a certain type or *looks* like it has a certain property. Customers will want to be sure they get the type of quadrilateral they ordered!

Discussion Points

What special properties might a quadrilateral have?

What algebra tools could be useful?

What types of quadrilaterals might be ordered?

The orders:

a. A quadrilateral formed by the intersection of these lines:

$y = \frac{-3}{2}x + 3$ $\qquad$ $y = \frac{3}{2}x - 3$ $\qquad$ $y = -\frac{3}{2}x + 9$ $\qquad$ $y = \frac{3}{2}x + 3$

b. A quadrilateral with vertices at these points:

A(0, 2) *B*(1, 0) *C*(7, 3) *D*(4, 4)

c. A quadrilateral with vertices at these points:

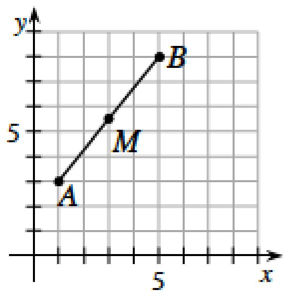
W(0, 5) *X*(2, 7) *Y*(5, 7) *Z*(5, 1)

METHODS AND MEANINGS

MATH NOTES

Finding a Midpoint

A **midpoint** is a point that divides a line segment into two parts of equal length. For example, *M* is the midpoint of $\overline{AB}$ below.



There are several ways to find the midpoint of a line segment if the coordinates of the endpoints are known. One way is to add half the change in

x ($\frac{1}{2}$ Δx) and half of the change in *y* ($\frac{1}{2}$ Δy) to the *x*-and *y*-coordinates of the starting point, respectively.

Thus, if A(1, 3) and *B*(5, 8), then $\Delta x = 5 - 1 = 4$ and $\Delta y = 8 - 3 = 5$. Then the *x*-coordinate of *M* is $1 + \frac{1}{2}(4) = 3$ and the *y*-coordinate is $3 + \frac{1}{2}(5) = 5.5$. So point *M* is at (3, 5.5).

This strategy can be used to find other points between *A* and *B* that are a proportion of the way from a starting point. For example, if you wanted to find a point $\frac{4}{5}$ of the way from point *A* to point *B*, then this could be found by adding $\frac{4}{5}$ of Δx to the *x*-coordinate of point *A* and adding $\frac{4}{5}$ of Δy to the *y*-coordinate of point *A*. This would be the point $(1 + \frac{4}{5}(4), 3 + \frac{4}{5}(5))$ which is (4.2, 7). Generally, a point a ratio *r* from $A(x_0, y_0)$ to $B(x_1, y_1)$ is at $(x_0 + r(x_1 - x_0), y_0 + r(y_1 - y_0))$.