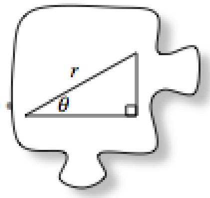
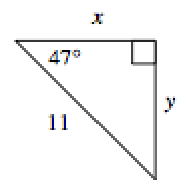
5.1.1 What if I know the hypotenuse?

Sine and Cosine Ratios



5-7. You now have multiple trig tools to find missing side lengths of triangles. For the triangle at right, find the values of x and y. Your Triangle Toolkit might help. Which tools did you use?



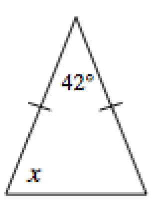
5-8. Lori has written the conjectures below. For each one, decide if it is true or not. If you believe it is not true, find a **counterexample** (an example that proves that the statement is false).

a. If a shape has four equal sides, it cannot be a parallelogram.

b. If tan θ is more than 1, then θ must be more than 45°.

c. If two angles formed when two lines are cut by a transversal are corresponding, then the angles are congruent.

5-9. Multiple Choice: In the triangle below, x must be:



a. 42°

b. 69°

c. 21°

d. 138°

e. none of these

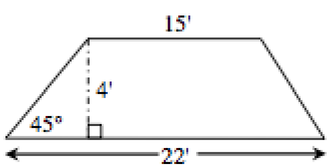
5-10. Susannah is drawing a card from a standard 52-card deck. See the entry "playing cards" in the glossary to learn what playing cards are included a deck.

a. What is the probability that she draws a card that is less than 5?

b. What is the probability that the card she draws is 5 or more? Use a complement.

c. What is the probability that the card she draws is a red card or a face card? Show how you can use the Addition Rule to determine this probability.

5-11. Find its area and perimeter of the trapezoid below. Keep your work organized so that you can later explain how you solved it. (Note: The diagram is not drawn to scale.)



5-12. Solve each of the equations below for the given variable. Be sure to check your answers.

a. $4(2x + 5) - 11 = 4x - 3$

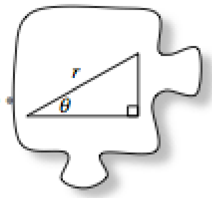
b. $\frac{2m-1}{19} = \frac{m}{10}$

c. $3p^2 + 10p - 8 = 0$

d. $\sqrt{x+2} = 5$

5.1.2 Which tool should I use?

Selecting a Trig Tool

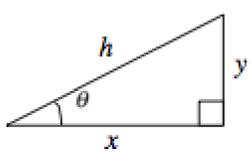


METHODS AND MEANINGS

MATH NOTES

Trigonometric Ratios

You now have three **trigonometric ratios** you can use to solve for the missing side lengths and angle measurements in any right triangle. In the triangle below, when the sides are described relative to th angle ϑ, the opposite leg is *y* and the adjacent leg is *x*. The hypotenuse is *h* regardless of which acute angle is used.

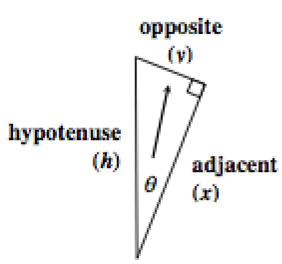


$$\tan\theta = \frac{\text{opposite leg}}{\text{adjacent leg}} = \frac{y}{x}$$

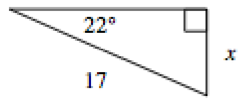
$$\sin\theta = \frac{\text{opposite leg}}{\text{hypotenuse}} = \frac{y}{h}$$

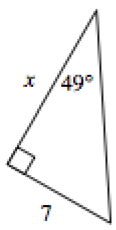
$$\cos\theta = \frac{\text{adjacent leg}}{\text{hypotenuse}} = \frac{x}{h}$$

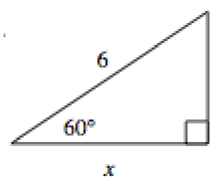
In some cases, you may want to rotate the triangle so that it looks like a slope triangle in order to easily identify the reference angle ϑ, the opposite leg *y*, the adjacent leg *x*, and the hypotenuse *h*. Instead of rotating the triangle, some people identify the opposite leg as the leg that is always opposite (not touching) the angle. For example, in the diagram below, *y* is the leg opposite angle ϑ.



5-17. For each triangle below, write an equation relating the reference angle (the given acute angle) with the two side lengths of the right triangle. Then solve your equation for *x*.

a. 

b. 

c. 

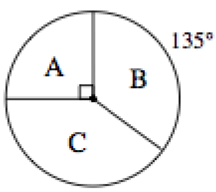
5-18. While shopping at his local home improvement store, Chen noticed that the directions for an extension ladder state, "*This ladder is most stable when used at a 75° angle with the ground.*" He wants to buy a ladder to paint a two-story house that is 26 feet high. How long does his ladder need to be? Draw a diagram and set up an equation for this situation. Show all work.

5-19. Examine each sequence below. State whether it is arithmetic, geometric, or neither. For the sequences that are arithmetic or geometric, find the formula for $t(n)$ or a_n.

a. 100, 10, 1, 0.1, …

b. 0, –50, –100, …

5-20. The spinner below has three regions: A, B, and C. To play the game, you must spin it *twice*. If the game were played 80 times, how many times would you expect to get A on both spins? Use a tree diagram or area model to help you answer the question. Test your ideas by creating the spinner below with the *Single Spinner Label eTool* and experimentally play the game.



5-21. Lori has written the conjectures below. For each one, decide if it is true or not. If you believe it is not true, find a counter example (an example that proves that the statement is false).

a. If a triangle has a 60° angle, it must be an equilateral triangle.

b. To find the area of a shape, you always multiply the length of the base by the height.

c. All shapes have 360° rotation symmetry.

5-22. Multiply each polynomial. That is, change each product to a sum.

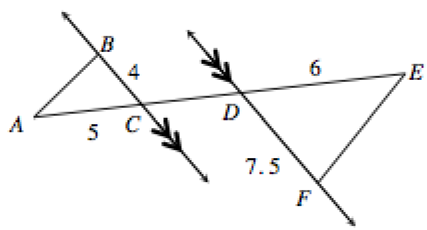
a. $(2x + 1)(3x - 2)$

b. $(2x + 1)(3x^2 - 2x - 5)$

c. $(3y - 8)(-x + y)$

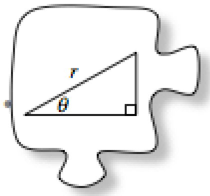
d. $(x - 3y)(x + 3y)$

5-23. Examine the triangles below. Are the triangles similar? If so, show how you know with a flowchart. If not, explain how you know they cannot be similar.



5.1.3 How can I find the angle?

Inverse Trigonometry



5-29. Solve the following equations for the given variable, if possible. Remember to check your answers.

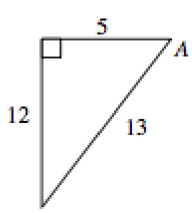
a. $6x^2 = 150$

b. $4m + 3 - m = 3(m + 1)$

c. $\sqrt{5x-1} = 3$

d. $(k - 4)^2 = -3$

5-30. Use two different trig ratios to find the measure of $\angle A$. Did you get the same answer both ways?



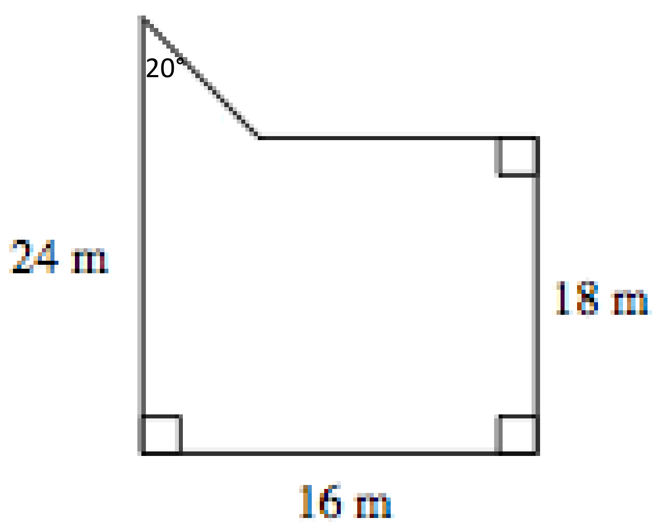
5-31. Mervin and Leela are in bumper cars. They are at opposite ends of a 100-meter track heading toward each other. If Mervin moves at a rate of 5.5 meters per second and Leela moves at a rate of 3.2 meters per second, how long does it take for them to collide?

5-32. Assume that two standard dice are being rolled. Let event A = {the sum is a multiple of 3} and event B = {the sum is a multiple of 4}. The $P(A) = \frac{12}{36}$ and the $P(B) = \frac{9}{36}$.

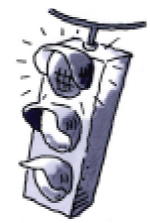
a. How many outcomes are in the intersection of events A and B?

b. What is P(A or B)?

5-33. Find the area and the perimeter of the figure below. Be sure to organize your work so you can explain your method later.

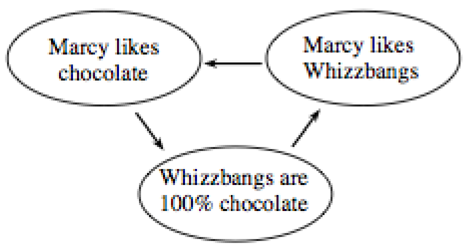


5-34. Jerry was trying to use a flowchart to describe how his friend Marcy feels about Whizzbangs candy. Examine his flowchart below.



a. How do you know Jerry's flowchart is incorrect?

b. Make a flowchart on your paper with the same three ovals, but with arrows drawn in so the flowchart makes sense. Explain why your flowchart makes more sense that the one below.



5-35. While playing a board game, Mimi noticed that she could roll the dice 8 times in 30 seconds. How many minutes should it take her to roll the dice 150 times?

5.1.4 How can I use trig ratios?

Trigonometric Applications

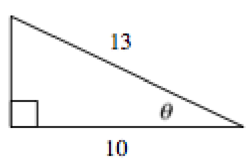
METHODS AND MEANINGS

MATH NOTES

Inverse Trigonometry

Just as subtraction "undoes" addition and multiplication "undoes" division, the inverse trigonometric functions "undo" the trigonometric functions tangent, sine, and cosine. Specifically, inverse trigonometric functions (**inverse cosine, inverse sine, and inverse tangent**) are used to find the measure of an acute angle in a right triangle when a ratio of two sides is known. This is the **inverse**, or opposite, of finding the trigonometric ratio from a known angle.

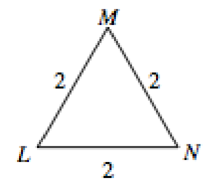
The inverse trigonometric functions that will be used in this course are: $\mathbf{sin^{-1}}$, $\mathbf{cos^{-1}}$, and $\mathbf{tan^{-1}}$ (pronounced "inverse sine," "inverse cosine" and "inverse tangent"). Below is an example that shows how $\cos^{-1}$ may be used to find a missing angle, θ.

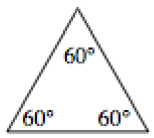


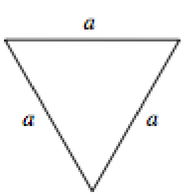
$$\cos\theta = \frac{10}{13}$$
$$\theta = \cos^{-1}\left(\frac{10}{13}\right)$$
$$\theta \approx 39.7^\circ$$

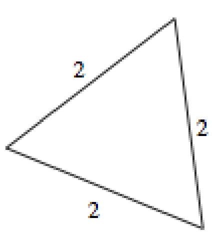
To evaluate $\cos^{-1}\left(\frac{10}{13}\right)$ on a scientific calculator, most calculators require the "2nd" or "INV" button to be pressed before the "cos" button.

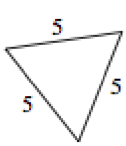
5-41. Which of the triangles below are similar to Δ*LMN* at right? How do you know? Explain.



a. 

b. 

c. 

d. 

5-42. Find the equation of the line that has a 33.7° slope angle and a *y*-intercept at (0, 7). Assume the line has a positive slope.

5-43. Write an equation for each sequence.

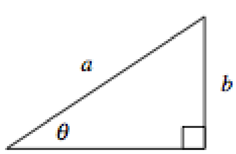
a. 108, 120, 132, …

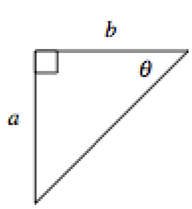
b. $\frac{2}{5}, \frac{4}{5}, \frac{8}{5}, \ldots$

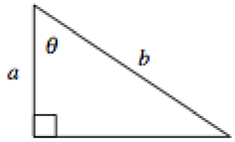
c. 3741, 3702, 3663, …

d. 117, 23.4, 4.68, …

5-44. For each triangle below, write a trigonometric equation relating *a*, *b*, and *ϑ*.

a. 

b. 

c. 

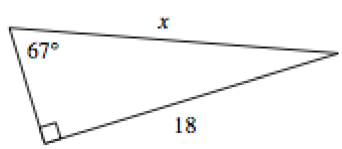
5-45. In a standard deck of 52 playing cards, 13 cards are clubs, and 3 of the clubs are "face" cards (K, Q, J). What is the probability of drawing one card that is:

a. A club or a face card? Is this a union or an intersection?

b. A club and a face card? Is this a union or an intersection?

c. Not a club and not a face card?

5-46. Estelle is trying to find *x* in the triangle below. She lost her scientific calculator, but luckily her teacher told her that sin 23º ≈ 0.391, cos 23º ≈ 0.921, and tan 23º ≈ 0.424.

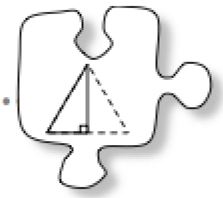


a. Write an equation that Estelle could use to solve for *x*.

b. Without a calculator, how could Estelle find sin 67°? Explain.

5.2.1 Is there a shortcut?

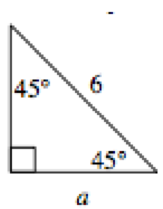
Special Right Triangles



LOOKING DEEPER

MATH NOTES

Rationalizing a Denominator



In Lesson 5.2.1, you developed some shortcuts to help find the lengths of the sides of a 30°- 60°- 90° and 45°- 45°- 90° triangle. This will enable you to solve similar problems in the future without a calculator or your Trig Table Toolkit.

However, sometimes using the shortcuts leads to some strange looking answers. For example, when finding the length of *a* in the triangle at right, you will get the expression $\frac{6}{\sqrt{2}}$.

A number with a radical in the denominator is difficult to estimate. Therefore, it is sometimes beneficial to **rationalize the denominator** so that no radical remains in the denominator. Study the example below.

Example: Simplify $\frac{6}{\sqrt{2}}$.

<u>Example</u>

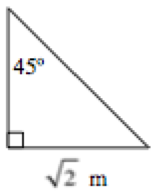
$$\frac{6}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{6\sqrt{2}}{2}$$
$$= 3\sqrt{2}$$

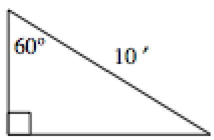
First, multiply the numerator and denominator by the radical in the denominator. Since $\frac{\sqrt{2}}{\sqrt{2}} = 1$, this does not change the value of the expression.

After multiplying, notice that the denominator no long has a radical, since $\sqrt{2} \cdot \sqrt{2} = 2$.

Often, the product can be further simplified. Since 2 divides evenly into 6, the expression $\frac{6\sqrt{2}}{2}$ can be rewritten as $3\sqrt{2}$.

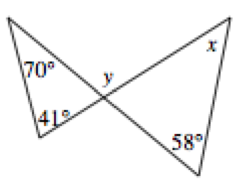
5-52. For each triangle below, use your triangle shortcuts from this lesson to find the missing side lengths. Then find the area and perimeter of the triangle.

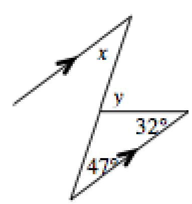
a. 

b. 

5-53. Use the relationships found in each of the diagrams below to solve for *x* and *y*. Assume the diagrams are not drawn to scale. State which geometric relationships you used.

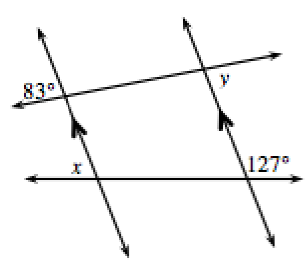
a.



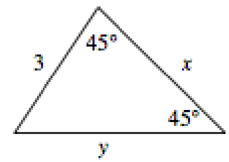


c.

b.



d.



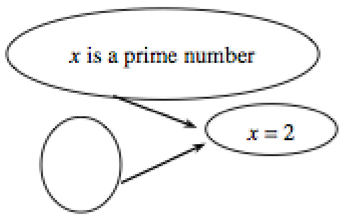
5-54. On graph paper, graph $\overline{AB}$ if $A(1, 6)$ and $B(5, 2)$.

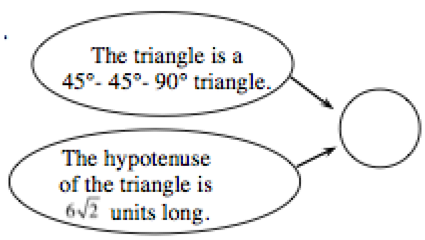
a. Find AB (the length of $\overline{AB}$). Leave your answer in exact form. That is, do not approximate with a decimal. Explain your method.

b. Reflect $\overline{AB}$ across the *y*-axis to create $\overline{A'B'}$. What type of shape is $ABB'A'$ if the points are connected in order? Then find the area of $ABB'A'$.

5-55. In a random sample of 10,000 college students, a research company found that 35.7% were involved in a club and 27.8% studied 4 or more hours per day. When they reported their findings, the research company indicated that 53.4% of college students were either involved in a club or they studied 4 or more hours per day. Given this information, what is the probability that a college student is involved in a club and studies 4 or more hours per day?

5-56. Fill in the blank ovals below so that each flowchart is correct.

a. 

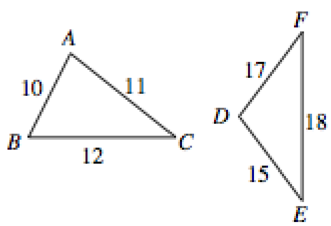
b. 

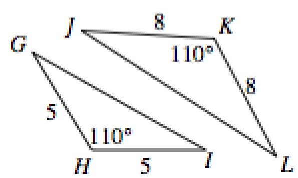
5-57. Write an explicit equation for the following sequences.

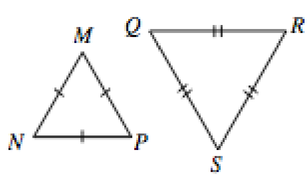
a. 500, 2000, 3500, …

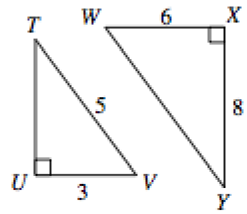
b. 30, 150, 750, 3750, …

5-58. Decide if each pair of triangles below are similar. If they are similar, give a sequence of rigid transformations that justifies your conclusion. If they are not similar, explain how you know.

a. 

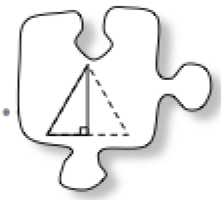
b. 

c. 

d. 

5.2.2 How can I use similar triangles?

Pythagorean Triples

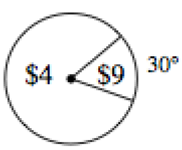


LOOKING DEEPER

MATH NOTES

Expected Value

The amount you would expect to win (or lose) per game after playing a game of chance many times is called the **expected value**. This value does not need to be a possible outcome of a single game, but instead reflects an average amount that will be won or lost per game.

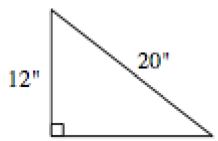


For example, the "$9" portion of the spinner at right makes up $\frac{30}{360} = \frac{1}{12}$ of the spinner, while the "$4" portion is the rest, or $\frac{11}{12}$, of the spinner. If the spinner was spun 12 times, probability predicts that it would land on "$9" once and "$4" eleven times. Therefore, someone spinning 12 times would expect to receive 1($9) + 11($4) = $53. On average, each spin would earn an expected value of $\frac{\$53}{12 \text{ spins}} \approx \$4.42$. You could use this value to predict the result for any number of spins. For example, if you play 30 times, you would expect to win 30($4.42) = $132.50.

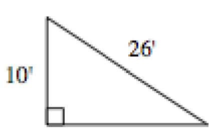
Another way to calculate expected value involves the probability of each possible outcome. Since "$9" is expected $\frac{1}{12}$ of the time, and "$4" is expected $\frac{11}{12}$ of the time, then the expected value can be calculated with the expression $(\$9)(\frac{1}{12}) + (\$4) + (\frac{11}{12}) = \frac{\$53}{12} \approx \$4.42$.

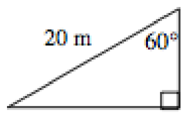
A **fair game** is one in which the expected value is zero. Neither player expects to win or lose if the game is played numerous times.

5-64. The sides of each of the triangles below can be found using one of the shortcuts from Section 5.2. Try to find the missing lengths using your patterns. Do not use a calculator.

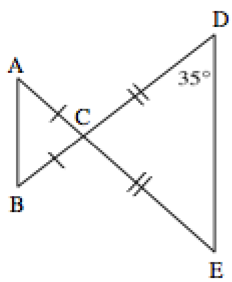
a. 

b. 

c. 

d. 

5-65. Copy the diagram below onto your paper.

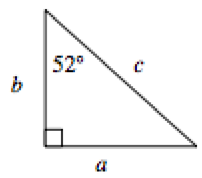


a. Find the measures of all the angles in the diagram.

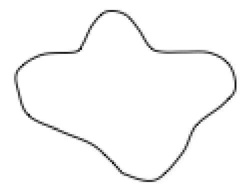
b. Make a flowchart showing that the triangles are similar.

c. Cheri and Roberta noticed their similarity statements for part (b) were not the same. Cheri had stated $\Delta ABC \sim \Delta DEC$, while Roberta maintained that $\Delta ABC \sim \Delta EDC$. Who is correct? Or are they both correct? Explain your reasoning.

5-66. Using the triangle below, write an expression representing $cos\ 52º$. Then write an expression for $tan\ 52º$ and $cos\ 38º$. What other ratio is equivalent to $cos\ 38º$?



5-67. Hadrosaurs, a family of duck-billed, plant-eating dinosaurs, were large creatures with thick, strong tails. It has recently been determined that hadrosaurs probably originated in North America.



Example of a hadrosaur footprint.

Scientists in Alaska recently found a hadrosaur footprint like the one at right that measured 14 inches across. It is believed that the footprint was created by a young dinosaur that was approximately 27 feet long. Adult hadrosaurs have been known to be 40 feet long. How wide would you expect a footprint of an adult hadrosaur to be? Show your reasoning.

5-68. A sequence starts –3, 1, 5, 9…

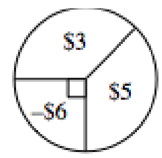
a. If you wanted to find the 50th term of the sequence, would an explicit equation or a recursive equation be more useful?

b. Write an equation that represents the sequence.

c. What is the 50^{th} term of the sequence?

d. Write an explicit equation for the sequence 3, $2\frac{2}{3}$, $2\frac{1}{3}$, 2, $1\frac{2}{3}$

5-69. The probability of winning \$3 on the spinner at right is equal to the chance of winning \$5. Find the expected value for one spin. Is this game fair?



5-70. Jeynysha has a Shape Bucket with a trapezoid, right triangle, scalene triangle, parallelogram, square, rhombus, pentagon, and kite. If she reaches in the bucket and randomly selects a shape, find:

a. P(at least one pair of parallel sides)

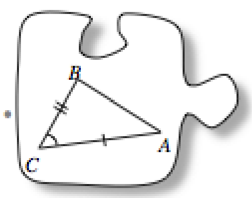
b. P(hexagon)

c. P(not a triangle)

d. P(has at least 3 sides)

5.3.1 What triangle tools do I still need?

Finding Missing Parts of Triangles



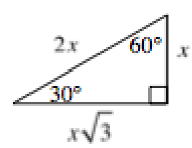
METHODS AND MEANINGS

MATH NOTES

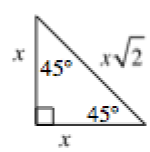
Special Right Triangles

So far in this chapter, you have learned about several special right triangles. Being able to recognize these triangles will enable you to quickly find the lengths of the sides and will save you time and effort in the future.

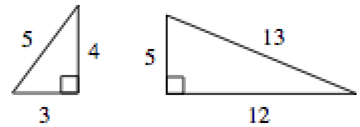
The half-equilateral triangle is also known as the **30°- 60°- 90° triangle**. The sides of this triangle are always in the ratio $1 : \sqrt{3} : 2$, as shown below.



Another special triangle is the **45°- 45°- 90° triangle**. This triangle is also commonly known as an isosceles right triangle. The ratio of the sides of this triangle is always $1 : 1 : \sqrt{2}$.



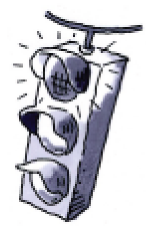
You also discovered several **Pythagorean Triples**. A Pythagorean Triple is any set of 3 positive integers *a*, *b*, and *c* for which $a^2 + b^2 = c^2$. Two of the common Pythagorean Triples that you will see throughout this course are shown below.

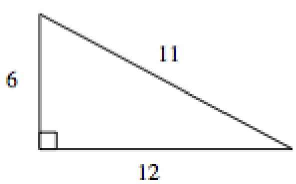


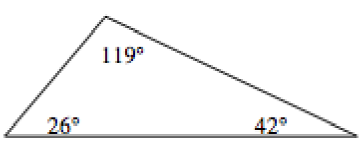
5-77. To paint a house, Travis leans a ladder against the wall. If the ladder is 16 feet long and it makes contact with the house 14 feet above ground, what angle does the ladder make with the ground? Draw a diagram of this situation and show all work.

5-78. WACKY DIAGRAMS

After drawing some diagrams on his paper, Jason thinks there is something wrong. Examine each diagram below and decide whether or not the triangle could exist. If it cannot exist, explain why not.



a. 

b. 

5-79. William thinks that the hypotenuse must be the longest side of a right triangle, but Chad does not agree. Who is correct? Support your answer with an explanation and a counterexample, if possible.

5-80. Plot ΔABC on graph paper with points $A(3, 3)$, $B(1, 1)$, and $C(6, 1)$.

a. Multiply all of the *y*-coordinates of ΔABC by –1. Then use the function $(x, y) \rightarrow (x - 6, y - 3)$ to translate the triangle. Name the coordinates of the result.

b. Rotate ΔABC 90° counterclockwise (↺) about the origin. Then reflect the result across the *y*-axis. Name the coordinates of the result.

5-81. Solve the equations below, if possible. If there is no solution, explain why.

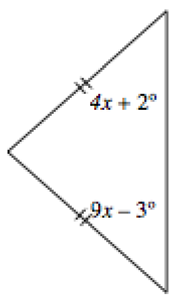
a. $\frac{8-x}{x} = \frac{3}{2}$

b. $-2(5x - 1) - 3 = -10x$

c. $x^2 + 8x - 33 = 0$

d. $\frac{2}{3}x - 12 = 180$

5-82. Multiple **Choice:** Based on the relationships provided in the diagram, which of the equations below is correct? Justify your solution.



a. $4x + 2° + 9x - 3° = 90°$

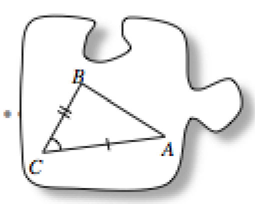
b. $4x + 2° = 9x - 3°$

c. $4x + 2° + 9x - 3° = 180°$

d. $(4x + 2°)(9x - 3°) = 90°$

5.3.2 Is there a faster way?

Law of Sines



METHODS AND MEANINGS

MATH NOTES

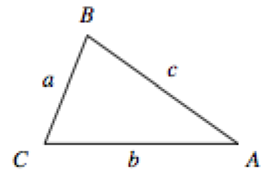
Law of Sines

For any ΔABC , the ratio of the sine of an angle to the length of the side opposite the angle is constant. This means that:

$$\frac{\sin(m\angle A)}{a} = \frac{\sin(m\angle B)}{b},$$

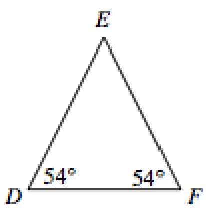
$$\frac{\sin(m\angle B)}{b} = \frac{\sin(m\angle C)}{c}, \text{ and}$$

$$\frac{\sin(m\angle A)}{a} = \frac{\sin(m\angle C)}{c}.$$



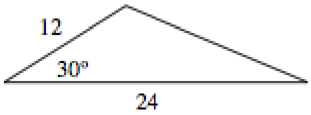
This property is called the **Law of Sines**. This is a powerful tool because you can use the sine ratio to solve for measures of angles and lengths of sides of *any* triangle, not just right triangles. The law works for angle measures between 0° and 180°.

5-89. Lizzie noticed that two angles in Δ*DEF*, shown below, have the same measure. Based on this information, what statement can you make about the relationship between $\overline{DE}$ and $\overline{EF}$?

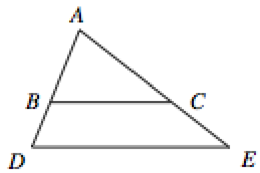


5-90. Find the length of $\overline{DF}$ in the diagram from problem 5-89 if DE = 9 mm.

5-91. Find the area of the triangle below. Show all work.



5-92. In the diagram below, ΔABC and ΔADE are similar. If $AB = 5$, $BD = 4$, and $BC = 7$, then what is DE?



5-93. Stephen does not like yogurt very much, but he loves apples. Since both make a good snack, Stephen's mom makes a deal with Stephen. She will keep the refrigerator stocked with 5 yogurts, 2 green apples, and 3 red apples every day. Each day, Stephen will randomly pick a snack. What is the probability Stephen will *not* get three yogurts on three consecutive days? Use a tree diagram or area model to show all the possible outcomes in the sample space.

5-94. Solve each equation below for *x*. Check your solution if possible.

a. $\frac{4}{5}x - 2 = 7$

b. $3x^2 = 300$

c. $\frac{4x-1}{2} = \frac{x+5}{3}$

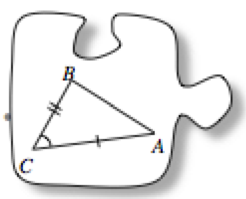
d. $x^2 - 4x + 6 = 0$

5-95. Solve the system of equations below. Write your solution as a point in (x, y) form. Check your solution.

$y = -3x - 2$
$2x + 5y = 16$

5.3.3 How can I complete my triangle toolkit?

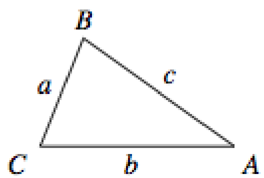
Law of Cosines



METHODS AND MEANINGS

MATH NOTES

Law of Cosines



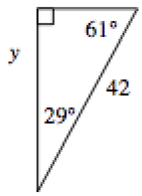
Just like the Law of Sines, the **Law of Cosines** represents a relationship between the sides and angles of a triangle.

Specifically, when given the lengths of any two sides, such as a and b, and the angle between them, $\angle C$, the length of the third side, in this case c, can be found using this relationship

$$c^{2} = a^2 + b^2 - 2ab\cos C$$

Similar equations can be used to solve for a and b.

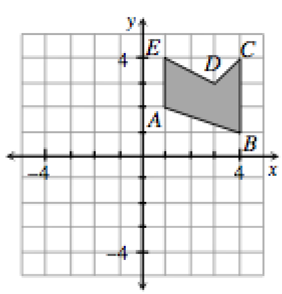
5-100. Eugene wants to use the cosine ratio to find y on this triangle.



a. Which angle should he use to write an equation and solve for y using the cosine ratio? Why?

b. Set up an equation, and solve for y using cosine.

5-101. Copy the graph below onto graph paper.



a. If the shape *ABCDE* were rotated about the origin 180°, where would point A' be?

b. If the shape *ABCDE* were reflected across the *x*-axis, where would point *C′* be?

c. If the shape *ABCDE* were translated so that each point (x, y) corresponds to $(x - 1, y + 3)$, where would point *B′* be?

5-102. On graph paper, graph the line $y = \frac{3}{4}x + 6$. Then find the slope angle (the acute angle the line makes with the *x*-axis).

5-103. An 8 foot ladder leaning against a house touches 7 feet above the ground. Draw a diagram and determine the measure of the angle created by the ladder and the ground.

5-104. This problem is a checkpoint for multiplying polynomials and solving quadratic equations. It will be referred to as Checkpoint 5A.

In parts (a) and (b) rewrite the expression without parentheses. In parts (c) and (d) solve each equation.

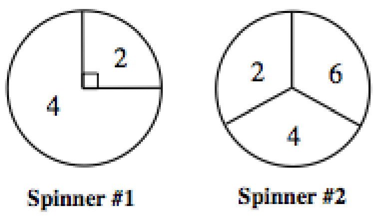
a. $2x(x + 3)$

b. $(3x + 2)(x - 3)$

c. $x^2 - 8x + 7 = 0$

d. $y^2 - 2y = 15$

5-105. The spinners below are spun and the results are added.

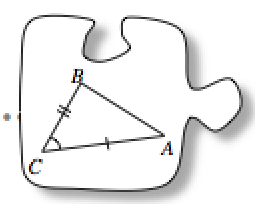


a. Find P(sum is 4).

b. Find P(sum is 8).

5.3.4 Is there more than one possible triangle?

Ambiguous Triangles



5-111. Farmer Jill has a problem. She lives on a triangular plot of land that is surrounded on all three sides by a fence. Yesterday, one side of the fence was torn down in a storm. She wants to determine the length of the side that needs to be rebuilt so she can purchase enough lumber. Since the weather is still too poor for her to go outside and measure the distance, she decides to use the lengths of the two sides that are still standing (116 feet and 224 feet) and the angle between them (58°).

a. Draw a diagram of this situation. Label all of the sides and angles that Farmer Jill has measurements for.

b. Find the length of the fence that needs to be replaced. Show all work. Which tool did you use?

5-112. Mr. Miller has informed you that two shapes are similar.

a. What does this tell you about the angles in the shapes?

b. What does this tell you about the lengths of the sides of the shapes?

5-113. Find the equation of the line that has a slope angle of 25° and a *y*-intercept of (0, 4). Sketch a graph of this line. Assume the slope of the line is positive.

5-114. Two sides of a triangle have lengths 9 and 14 units. Describe what you know about the length of the third side.

5-115. How many terms are in the arithmetic sequence shown below?

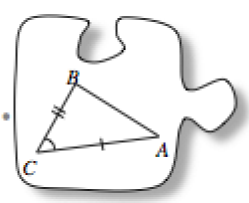
15, 7, –1, –9, ..., –225

5-116. Mr. Kyi has placed 3 red, 7 blue, and 2 yellow beads in a hat. If a person selects a red bead, he or she wins $3. If that person selects a blue bead, he or she loses $1. If the person selects a yellow bead, he or she wins $10. What is the expected value for one draw? Is this game fair?

5-117. Tehachapi High School has 839 students and is increasing by 34 students per year. Meanwhile, Fresno High School has 1644 students and is decreasing by 81 students per year. In how many years will the two high schools have the same number of students? Be sure to show all work.

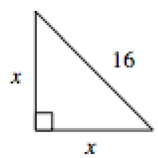
5.3.5 Which tool should I use?

Choosing a Tool



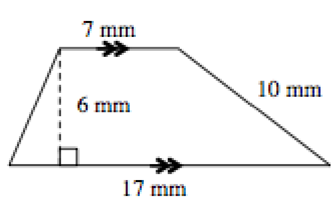
5-126. Solve for the missing side lengths and angles in the triangle below.

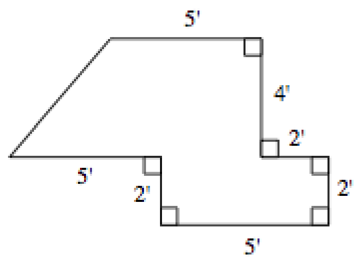
5-127. Examine the triangle shown below. Solve for *x twice*, using two different methods. Show your work for each method clearly.



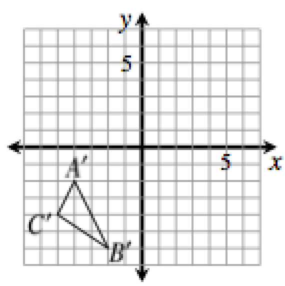
5-128. In Chapter 1 you learned that all rectangles are parallelograms because they all have two pairs of opposite parallel sides. Does that mean that all parallelograms are rectangles? Why or why not? Support your statements with reasons.

5-129. Find the area and perimeter of each shape below. Show all work.

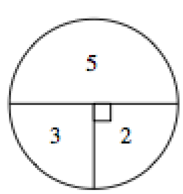
a. 

b. 

5-130. ΔABC was reflected across the *x*-axis, and then that result was rotated 90° clockwise about the origin to result in $\Delta A'B'C'$, shown below. Find the coordinates of points *A*, *B*, and *C* of the original triangle.

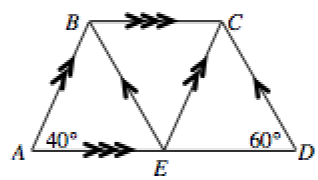


5-131. Marina needs to win 10 tickets to get a giant stuffed panda bear. To win tickets, she throws a dart at the dartboard below and wins the number of tickets listed in the region where her dart lands. Unfortunately, she only has enough money to play the game three times. If she throws the dart randomly, do you expect that she will be able to win enough tickets? Assume that each dart will land on the dartboard.



5-132. Find the equation of a line parallel to the line $y = \frac{3}{4}x - 5$ that passes through the point (−4, 1). Show how you found your answer.

5-133. Examine trapezoid *ABCD* below.

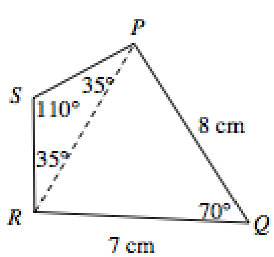


a. Find the measures of all the angles in the diagram.

b. What is the sum of the angles that make up the trapezoid *ABCD*? That is, what is $m\angle A + m\angle ABC + m\angle BCD + m\angle D$?

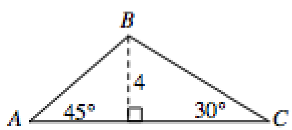
5-134. Use your triangle tools to solve the problems below.

a. Find *PR* in the diagram below.



b. Find the perimeter of quadrilateral *PQRS*.

5-135. Find the area and perimeter of Δ*ABC* below. Give approximate (decimal) answers, not exact answers.



5-136. This problem is a checkpoint for writing equations for arithmetic and geometric sequences. It will be referred to as Checkpoint 5B.

a. Write an explicit and recursive rule for $t(n) = 1, 4, 7, 10, \ldots$

b. Write an explicit and recursive rule for $t(n) = 3, \frac{3}{2}, \frac{3}{4}, \frac{3}{8}$

In parts (c) and (d), write an explicit rule for the sequence given in the $n \rightarrow t(n)$ tables.

c. An arithmetic sequence

n	$t(n)$
1	17
2	
3	3
4	

d. A geometric sequence

n	$t(n)$
1	
2	7.2
3	8.64
4	

e. If an arithmetic sequence has $t(7) = 1056$ and $t(12) = 116$, what is $t(4)$?

5-137. Bob is hanging a swing from a pole high off the ground so that it can swing a total angle of 120°. Since there is a bush 5 feet in front of the swing and a shed 5 feet behind the swing, Bob wants to ensure that no one will get hurt when they are swinging. What is the maximum length of chain that Bob can use for the swing?

a. Draw a diagram of this situation.

b. What is the maximum length of chain that Bob can use? State what tools you used to solve this problem.

5-138. On graph paper, draw ΔABC if $A(3, 2)$, $B(-1, 4)$, and $C(0, -2)$.

a. Find the perimeter of ΔABC .

b. Dilate ΔABC from the origin by a factor of 2 to create $\Delta A'B'C'$. What is the perimeter of $\Delta A'B'C'$?

c. If ΔABC is rotated 90° clockwise (↻) about the origin to form $\Delta A''B''C''$, name the coordinates of C''.

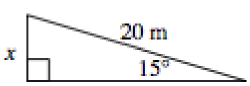
WHAT HAVE I LEARNED?

Most of the problems in this section represent typical problems found in this chapter. They serve as a gauge for you. You can use them to determine which types of problems you can do well and which types of problems require further study and practice. Even if your teacher does not assign this section, it is a good idea to try these problems and find out for yourself what you know and what you still need to work on.

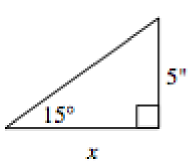
Solve each problem as completely as you can. The table at the end of the closure section has answers to these problems. It also tells you where you can find additional help and practice with problems like these.

CL 5-139. For each diagram, write an equation and solve to find the value for each variable.

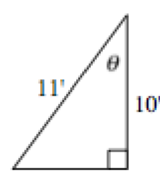
a.



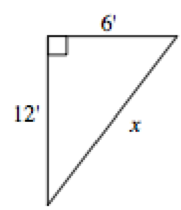
b.



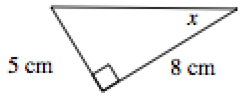
c.



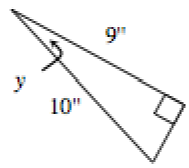
d.



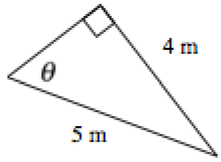
e.



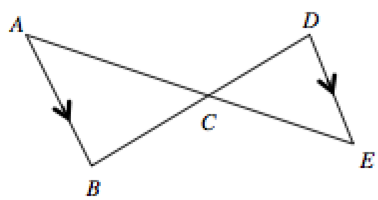
g.



f.



CL 5-140. Copy the diagram below onto your paper.

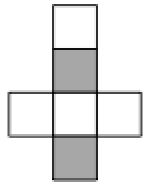


a. Are the triangles similar? If so, show your reasoning with a flowchart.

b. If $m\angle B = 80°$, $m\angle ACB = 29°$, $AB = 14$, and $DE = 12$, find CE .

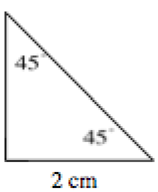
CL 5-141. STEP RIGHT UP!

At a fair, Cyrus was given the following opportunity. He could roll the die formed by the net at right one time. If the die landed so that a shaded die faced up, then Cyrus would win $10. Otherwise, he would lose $5. Is this game fair? Explain how you know.

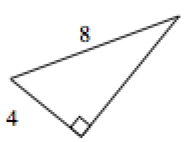


CL 5-142. Use your knowledge of special right triangles to find the missing side lengths and angle measures exactly. No calculators are needed.

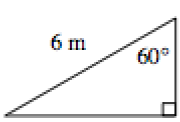
a.



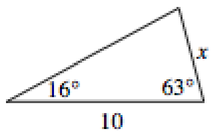
b.



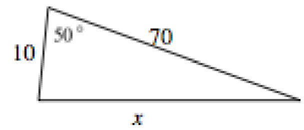
c.



CL 5-143. While working on homework, Zachary was finding the value of each variable in the diagrams below. His first step for each problem is shown under the diagram. If his first step is correct, continue solving the problem to find the solution. If his first step is incorrect, explain his mistake and solve the problem correctly.

a. 

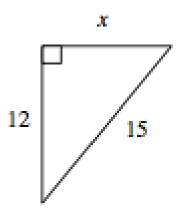
$$\sin 16° = \frac{x}{10}$$

b. 

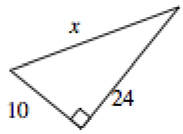
$x^2 = 10^2 + 70^2 - 2(10)(70)\cos 50°$

CL 5-144. In parts (a) and (b), use what you know about Pythagorean Triples to find the third side quickly. In parts (c) and (d), give all possible lengths for the third side of the triangle.

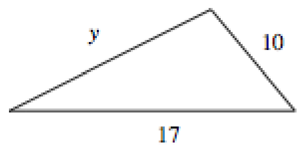
a.



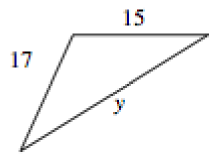
b.



c.



d.

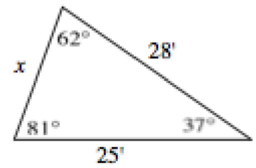


CL 5-145. Kayla brought snacks for her and her partner on the volleyball team. She packed flavored water (2 berry and 4 citrus), fruit (5 apricots, 2 apples, and 3 bunches of grapes), and small packages of crackers (2 regular and 2 whole wheat). Kayla will randomly choose one flavored water, one fruit, and one package of crackers.

a. Show all the possible combinations of three snacks that Kayla could choose.

b. What is the probability that Kayla will choose a high-fiber snack (any combination that includes both an apple and whole-wheat crackers)?

CL 5-146. Examine the triangle below. Solve for *x* *twice* using two different methods.



CL 5-147. Graph the points (3, –4) and (7, 2) on graph paper and draw the line segment and a slope triangle that connects the points.

Find:

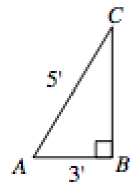
a. The length of the segment

b. The slope of the line segment

c. The area of the slope triangle

d. The measure of the slope angle

CL 5-148. Trace the figure below onto your paper and then perform all of the transformations listed below on the same diagram. Then find the perimeter of the final shape.



- Reflect ΔABC across $\overline{AB}$.
- Rotate ΔABC 180° around the midpoint of $\overline{BC}$.
- Reflect ΔABC across $\overline{AC}$.

5-149. A snack cracker company surveyed 1000 people, in different age groups, to determine their favorite cracker.

Age of Participants (years)

	Under 20	20 to 39	40 to 59	60 and over
# people	250	250	250	250

Favorite Cracker

	Cracker A	Cracker B	Cracker C
# people	371	308	321

a. What is the probability (represented as a percent) that a randomly selected participant was 20 years old or older?

b. 152 of the participants under 20 years old chose cracker A as their favorite. Calculate the probability that a participant chose cracker A *or* was under 20 years old.

c. What is the probability that a participant did not choose cracker A *and* was 20 or over years old? Show how you used a complement to answer this problem.

CL 5-150. Check your answers using the table at the end of this section. Which problems do you feel confident about? Which problems were hard? Have you worked on problems like these in math classes you have taken before? Use the table to make a list of topics you need help on and a list of topics you need to practice more.