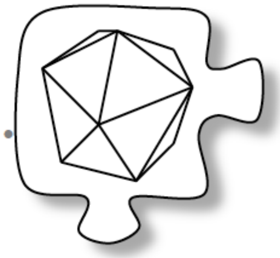
11.1.2 How can I measure it?



Pyramids

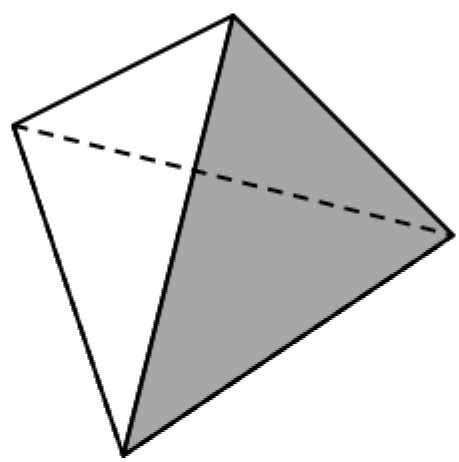
In Lesson 11.1.1, you explored Plato's five special solids: the tetrahedron, the octahedron, the cube (also known as the hexahedron), the dodecahedron, and the icosahedron. You discovered why these are the only regular polyhedra and developed a method to find their surface area.

Today you will examine the tetrahedron from a new perspective: as a member of the **pyramid** family. As you work today with your team, you will discover ways to classify pyramids by their shape and will develop new tools of measurement.

11-22. A **pyramid** is a polyhedron with a polygonal base formed by connecting each point of the base to a single given point (the **apex**) that is above or below the flat surface containing the base. Each triangular lateral face of a pyramid is formed by the segments from the apex to the endpoints of a side of the base and the side itself. A tetrahedron is a special pyramid because any face can act as its base.

Obtain the four Lesson 11.1.2 Resource Pages, a pair of scissors, and either tape or glue from your teacher. Have each member of your team build one of the solids. When assembling each solid, be sure to have the printed side of the net on the exterior of the pyramid for reference later. Then answer the questions below.

a. Sketch each pyramid onto your paper. What is the same about each pyramid? What is different? With your team, list as many qualities as you can.



b. A tetrahedron can also be called a **triangular-based pyramid**, because its base is always a triangle. Choose similar, appropriate names for the other pyramids that your team constructed.

c. Find the surface area of pyramids B and D. Use a ruler to find the dimensions of the edges in centimeters.

d. Compare pyramids B and C. Which do you think has more volume? Justify your reasoning.

11-23. THE TRANSAMERICA BUILDING

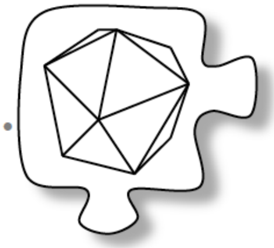
The TransAmerica building in San Francisco is built of concrete and is shaped like a square-based pyramid. The building is periodically power-washed using one gallon of cleaning solution for every 250 square meters of surface. As the new building manager, you need to order the cleaning supplies for this large task. The problem is that you do not know the height of each triangular face of the building; you only know the vertical height of the building from the base to the top vertex.

Your Task: Determine the amount of cleaning solution needed to wash the TransAmerica building if an edge of the square base is 96 meters and the height of the building is 220 meters. Include a sketch in your solution.

11-24. Read the Math Notes box for this lesson, which introduces new vocabulary terms such as **slant height** and **lateral surface area**. Explain the difference between the slant height and the height of a pyramid. How can you use one to find the other?

11.1.3 What is the volume?

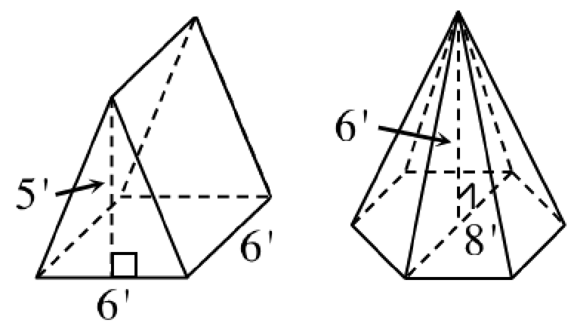
Volume of a Pyramid



Today, as you continue your focus on pyramids, look for and utilize connections to other geometry concepts. The models of pyramids that you constructed in Lesson 11.1.2 will be useful as you develop a method for finding the volume of a pyramid.

11-33. GOING CAMPING

As Soraya shopped for a tent, she came across two models that she liked best, shown at right. However, she does not know which one to pick! They are both made by the same company and appear to have the same quality. She has come to you for help in making her decision.



Tent A Tent B

While she says that her drawings are not to scale, below are her notes about the tents:

Tent A is a pup tent sitting on one of its rectangular faces. It has a triangular base. Its height is 5 feet, and its length and width are both 6 feet.

Tent B is a 6-foot-tall teepee. Its base is a regular hexagon, and the greatest diagonal across the floor measures 8 feet.

With your team, discuss the following questions in any order. Be prepared to share your discussion with the class.

- What are the shapes of the two tents?
- Without doing any calculations, which tent do you think Soraya should buy and why?
- What types of measurement might be useful to determine which tent is better?
- What do you still need to know to answer her question?

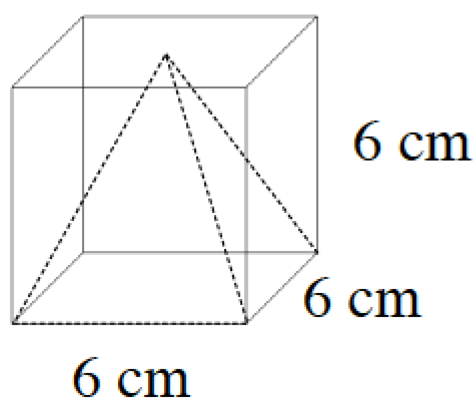
11-34. COMPARING SOLIDS

To analyze Tent B from problem 11-33, you need to know how to find the volume of a pyramid. But how can you find that volume?

To start, consider a simpler pyramid with a square base, such as pyramid B that your team built in Lesson 11.1.2. To develop a method to find the volume of a pyramid, first consider what solids(s) you could compare it to. For example, when finding the area of a triangle, you compared it to the area of a rectangle and figured out that the area of a triangle is always half the area of a rectangle with the same base and height. To what solids(s) can you compare the volume of pyramid B? Discuss this with your team and be prepared to share your thinking with the class.

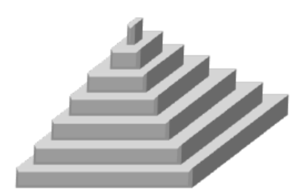
11-35. VOLUME OF A PYRAMID

Soraya thinks that pyramid B could be compared to a cube, like the one shown at right, since the base edges and heights of both are 6 cm.

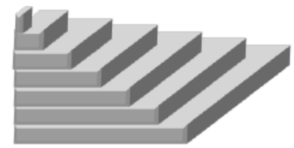


a. First estimate. What proportion of the cube is pyramid B? Discuss this with your team.

b. Soraya remembers comparing pyramids B and C in Lesson 11.1.2. She decided to compare the volumes by thinking of it as a stack of slices. When thinking of it this way, what is the shape of each layer? Note: The name for the shape of a layer of a three-dimensional solid is called a **cross-section**.



c. Soraya then slid all of the layers of the pyramid so that the top vertex was directly above one of the corners of the base, like Pyramid C from problem 11-22.



When the top vertex of a pyramid is directly above (or below) the center of the base, the pyramid is called a **right pyramid**, while all other pyramids are referred to as **oblique pyramids**.

When Soraya slid the layers to create an oblique pyramid, she did not add or take away any foam layers. How does the volume of her oblique pyramid compare with the right pyramid in part (b) above?

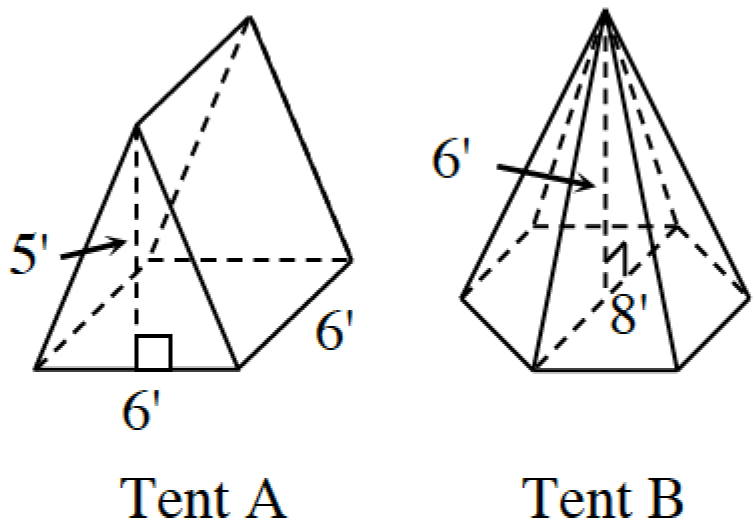
d. Test your estimate from part (a) by using as many copies of pyramid C as you need to assemble a cube. Was your estimate accurate? Now explain how to find the volume of a pyramid.

11-36. In problem 11-35, you may have noticed that the special square-based pyramid had one-third the volume of the cube. It turns out that this relationship between a pyramid and a prism with the same base area and height works for all other pyramids as well.

a. Write an expression for the volume of a pyramid with base area B and height h.

b. Use your expression from part (a) to find the volume of a pyramid with base area of 34 square units and height of 9 units.

11-37. Now return to problem 11-33 and help Soraya decide which tent to buy for her backpacking trip. To make this decision, compare the volumes, base areas, and surface areas of both tents. Be ready to share your decision with the class.



11-38. THREE-DIMENSIONAL SOLIDS TOOLKIT

Obtain the Lesson 11.1.3A Resource Pages ("Three-Dimensional Solids Toolkit") from your teacher. On the Resource Page, write everything you know about finding the volume and surface area of all of the solids that you have studied so far. In later lessons, you will continue to add information to this toolkit, so be sure to keep this resource page in a safe place. At this point, your Toolkit should include:

Prisms Cylinders Pyramids

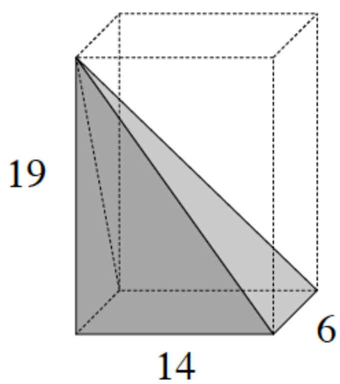
11.1.4 What if it is a cone?

Surface Area and Volume of a Cone

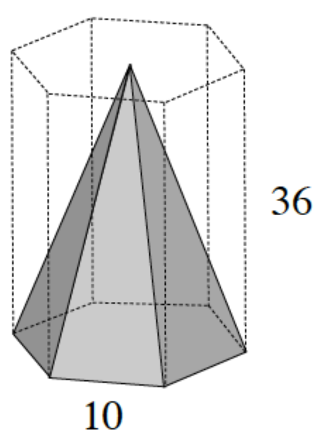
Today you will continue to use what you know about the volume and surface area of prisms and pyramids and will extend your understanding to include a new three-dimensional shape: a cone. As you work with your team, look for connections to previous course material.

11-53. Review what you learned in Lesson 11.1.3 by finding the volume of each pyramid below. Assume that the pyramid in part (a) corresponds to a rectangular-based prism and that the base of the pyramid and prism in part (b) is a regular hexagon.

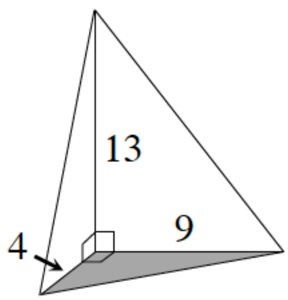
a.



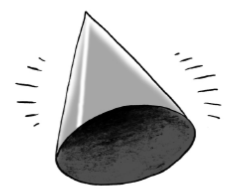
b.



c.



11-54. While finding the volumes of the pyramids in problem 11-53, Jamal asked, *"But what if it is a cone? How would you find its volume?"* Note that a **cone** is somewhat like a pyramid, but it has a circular base**.** Every point on the perimeter of the circular base connects to a point above the base called the apex.

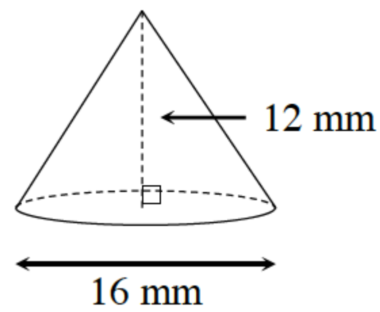


a. Discuss Jamal's question with your team. How might you use what you learned about the volume of pyramids to reason about the volume of a cone?

b. Lekili said, "*Remember when we found the area of a circle by finding what happens to the area of a regular polygon and the number of sides increase? I think we can use that approach here."* What do you think Lekili means? Explain how this can help find a method to compute the volume of a cone.

c. Use your ideas from part (b) to write an expression for the volume of a cone with radius of length r and height h.

d. Use your expression from part (c) to find the volume of a cone at right. Show all work.



11-55. HAPPY BIRTHDAY!

Your class has decided to throw your principal a surprise birthday party tomorrow. The whole class is working together to create party decorations, and your team has been assigned the job of producing party hats. Each party hat will be created out of special decorative paper and will be in the shape of a cone.

Your Task: Use the sample party hat provided by your teacher to determine the size and shape of the paper that forms the hat. Then determine the amount of paper (in square inches) needed to produce one party hat and figure out the total amount of paper you will need for each person in your class to have a party hat.

11-56. The Math Club has decided to sell giant waffle ice-cream cones at the Spring Fair. Lekili bought a cone, but then he got distracted. When he returned to the cone, the ice cream had melted, filling the cone to the very top!

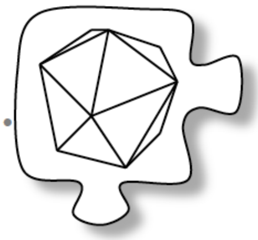
If the diameter of the base of the cone is 4 inches and the slant height is 6 inches, find the volume of the ice cream and the area of the waffle that made the cone.

11-57. THREE-DIMENSIONAL SOLIDS TOOLKIT

Add details to your Lesson 11.1.3A Resource Pages ("Three-Dimensional Solids Toolkit") for finding the volume and surface area of cones.

11.1.5 What is the relationship?

Surface Area and Volume of a Sphere



This lesson will complete your Three-Dimensional Solids Toolkit. You will learn about a new shape that you encounter often in your daily life: a sphere. You will also make connections between a cylinder, cone, and sphere of the same radius and height.

As you work with your team, keep the following focus questions in mind:

What's the relationship?

What other tools or information do I need?

11-66. Alonzo was blowing bubbles to amuse his little sister. He wondered, "*Why are bubbles always perfectly round?"*

a. Discuss Alonzo's question with the class. Why are free-floating bubbles always shaped like a perfectly round ball?

b. The shape of a bubble is called a **sphere**. What other objects can you remember seeing that are shaped like a sphere?

c. What shapes are related to spheres? How are they related?

11-67. GEOGRAPHY LESSON, Part One

Alonzo learned in his geography class that about 70% of the Earth's surface is covered in water. *"That's amazing!"* he thought. This information only made him think of new questions, such as "*What is the area of land covered in water?", "What percent of the Earth's surface is the United States?",* and "*What is the volume of the entire Earth?"*

Discuss Alonzo's questions with your team. Decide:

- What facts about the Earth would be helpful to know?
- What do you still need to learn to answer Alonzo's questions?

11-68. In order to answer his questions, Alonzo decided to get out his set of plastic geometry models. He has a sphere, cone, and cylinder that each has the same radius and height.

a. Draw an example diagram of each shape.

b. If the radius of the sphere is r, what is the height of the cylinder? How do you know?

c. Alonzo's models are hollow and are designed to hold water. Alonzo was pouring water between the shapes, comparing their volumes. He discovered that when he poured the water in the cone and the sphere into the cylinder, the water filled up the cylinder without going over! Determine what the volume of the sphere must be if the radius of the sphere is r units. Show all work.

11-69. Now that Alonzo knows that spheres, cylinders, and cones with the same height and radius are related, he decides to examine the surface area of each one. As he paints the exterior of each shape, he notices that the lateral surface area of the cylinder and the surface area of the sphere take exactly the same amount of paint! If the radius of the sphere and cylinder is r, what is the surface area of the sphere?

11-70. GEOGRAPHY LESSON, Part Two

Now that you have strategies for finding the volume and surface area of a sphere, return to problem 11-67 and help Alonzo answer his questions. That is, determine:

- The area of the Earth's surface that is covered in water.
- The percent of the Earth's surface that lies in the United States.
- The volume of the entire Earth.

Remember that in Chapter 10, you determined that the radius of the Earth is about 4,000 miles. Alonzo did some research and discovered that the land area of the United States is approximately 3,537,438 square miles.

11-71. THREE-DIMENSIONAL SOLIDS TOOLKIT

Retrieve the Three-Dimensional Solids Toolkit. Complete the entry for a sphere. That is, write everything you know about finding the volume and surface area of spheres. Be sure you include the relationships between the volumes of a cone, cylinder, and sphere with the same radius and height.