PC – Trig   Name KEY

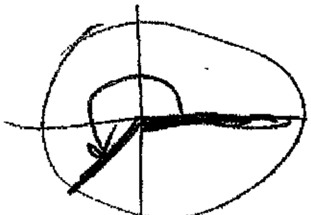
Review – The Unit Circle

1. Fill in the following table showing work to find Revolutions, Degrees, and Radians.

Revolutions	Degrees	Radians	Terminal Point (x , y)	Reference Angle
Below would be questions on the ~~Non~~-calculator portion				
$\frac{2}{3}$	$\frac{2}{3}\cdot 360 = 240°$	$\frac{2}{3}\cdot 2\pi = \frac{4\pi}{3}$	$\left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$	$\pi/3$
$\frac{3}{4}$	270°	$\frac{3\pi}{2}$	$(0, -1)$	$\pi/2$
$\frac{\frac{3\pi}{4}}{2\pi} = \frac{3}{8}$	135°	$\frac{3\pi}{4}$	$\left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$	$\pi/4$
$\frac{\frac{7\pi}{4}}{2\pi} = \frac{7}{8}$	315°	$\frac{7\pi}{4}$	$\left(\frac{\sqrt{2}}{2}, \frac{-\sqrt{2}}{2}\right)$	$\pi/4$
$\frac{1}{12}$	30°	$\frac{\pi}{6}$	$\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$	$\pi/6$
$\frac{1}{2}$	180°	π	(-1 , 0)	0
Below would be questions on the calculator portion				
$\frac{165}{360} = \frac{11}{24}$	165°	$\frac{165\pi}{180} = \frac{11\pi}{12}$	Approx. w/ (cos θ, sin θ) $(-0.966, 0.259)$	$\pi/12$
$\frac{4.5}{2\pi} = \frac{9}{4\pi}$	$\frac{4.5\cdot 180}{\pi} = 257.83°$	4.5 radians	Approx. w/ (cos θ, sin θ) $(-0.211, -0.978)$	$4.5 - \pi$
$\frac{7}{11}$	$\frac{7}{11}\cdot 360 = 229.1°$	$\frac{7}{11}\cdot 2\pi = \frac{14\pi}{11}$	Approx. w/ (cos θ, sin θ) $(-0.65, -0.756)$	$\frac{3\pi}{11}$

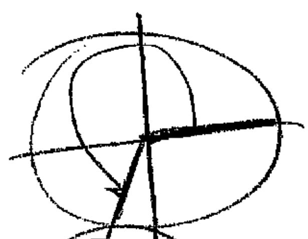
2. Picture an ant traveling along a unit circle in a **positive direction**, starting at the point (1,0). If the ant ends up at the point given determine how far did the ant traveled **in radians**? Consider using our paper "Convenient Coordinates". **Non-calculator**.

a. $\left(\frac{-\sqrt{2}}{2}, \frac{-\sqrt{2}}{2}\right)$



$\frac{5\pi}{4}$

b. $\left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$



$\frac{4\pi}{3}$

3. Is this point $\left(\frac{5}{7}, \frac{2\sqrt{6}}{7}\right)$ on the unit circle? Justify your answer! **Non-calculator**.

$\left(\frac{5}{7}\right)^2+\left(\frac{2\sqrt{6}}{7}\right)^2=\frac{25}{49}+\frac{12}{49}=\frac{47}{49}\neq 1$ *Not on the Unit Circle.

4. Suppose that you've traveled $^{19}\!/_6$ radians from standard position around a ferris wheel. Have you traveled more or less than half way around the Ferris wheel? Justify your answer. **Non-calculator**.

$\frac{19}{6}=3.1\overline{6}$, so this lager than π (≈ 3.14). The Ferris wheel traveled more than half-way around.

5. Determine if the following pairs of angles are coterminal or not. Show your work. **Calculator**.

a) 165° and -195°

$-195+360=165$

So *yes, coterminal

b) 845° and 7320°

So $845+360+360\ldots$??

$845+360x=7320$ (solve!) If whole #, then coterminal!

$\frac{360x}{360}=\frac{6475}{360}$

$x=17.99$ (*Not coterminal)

c) -45° and $\frac{\pi}{8}$

Convert to same measure! Coterminal angles are 2π apart

$-\frac{\pi}{4}+2\pi$

$-\frac{\pi}{4}+\frac{8\pi}{4}=\frac{7\pi}{4}$ (not the same as $\frac{\pi}{8}$)

*Not coterminal

6. Assume the point P is on the unit circle. Find P (x, y) from the given information. Show all your work and write your coordinates as completely simplified fraction. **Non-calculator**.

a. The x-coordinate is $\frac{5}{7}$ and point P is in quadrant IV.

$\left(\frac{5}{7}\right)^2+y^2=1$

$\frac{25}{49}+y^2=\frac{49}{49}$

$y^2=\frac{24}{25}$

$y=\frac{\sqrt{24}}{5}=\frac{2\sqrt{6}}{5}$

*$\left(\frac{5}{7}, -\frac{2\sqrt{6}}{5}\right)$

b. The x-coordinate is $\frac{-\sqrt{6}}{3}$ and point P is in the third quadrant.

$\left(-\frac{\sqrt{6}}{3}\right)^2+y^2=1$

$\frac{6}{9}+y^2=\frac{9}{9}$

$\sqrt{y^2}=\sqrt{\frac{3}{9}}$

$y=\frac{\sqrt{3}}{3}$

*$\left(-\frac{\sqrt{6}}{3}, -\frac{\sqrt{3}}{3}\right)$

8. Two angles in a triangle have radian measure $\frac{2\pi}{5}$ and $\frac{\pi}{3}$, What is the radian measure of the third angle? Show all work. **Non-calculator**.

Sum of angles $=180°$ or π radians

$\frac{2\pi}{5}+\frac{\pi}{3}+x=\pi \longrightarrow \frac{6\pi}{15}+\frac{5\pi}{15}+x=\frac{15\pi}{15}$ common denominators

3rd angle $\longrightarrow \boxed{x=\frac{4\pi}{15}}$

*Definitely Calculator! Oops!

Find one positive and one negative coterminal angle. ~~Non~~-calculator. Below are some examples, obviously not all!!

9) 1050° ± 360

$690°, 330°, -30°$

10) $-\frac{89\pi}{18} + \frac{2\pi}{1}\cdot\frac{18}{18}$

$-\frac{89\pi}{18} + \frac{36\pi}{18} = -\frac{53\pi}{18} + \frac{36\pi}{18} = -\frac{17\pi}{18} + \frac{36\pi}{18}$

$\boxed{= \frac{19\pi}{18}}$

11) 240° $+360 = 600°$

-360

$-120°$

12) $-\frac{14\pi}{9} + 2\pi\cdot\frac{9}{9}$

$-\frac{14\pi}{9} + \frac{18\pi}{9} = \frac{4\pi}{9}$

$-\frac{14\pi}{9} - \frac{18\pi}{9} = -\frac{32\pi}{9}$

13) $\frac{103\pi}{36} \pm \frac{2\pi}{1}\cdot\frac{36}{36}$

$\frac{103\pi}{36} - \frac{72\pi}{36} = \frac{31\pi}{36} - \frac{72\pi}{36}$

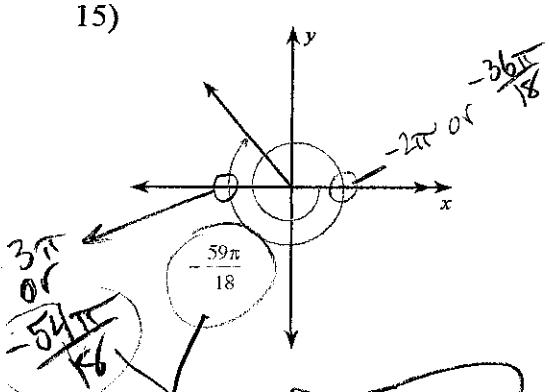
$= -\frac{41\pi}{36}$

14) -960° ± 360

$-960 + 360 = -600 + 360\cdot 2 = 120$

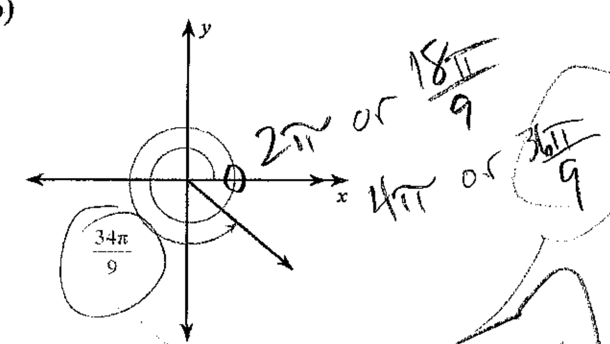
Find the exact reference angle in radians. Non-calculator

15)



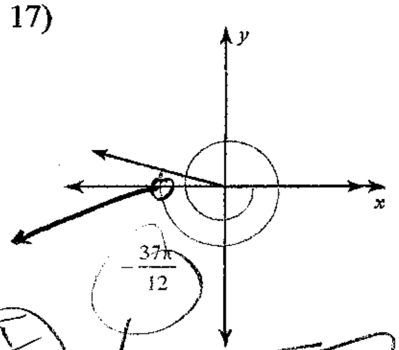
$RA = \boxed{\frac{5\pi}{18}}$

16)



$RA = \boxed{\frac{2\pi}{9}}$

17)

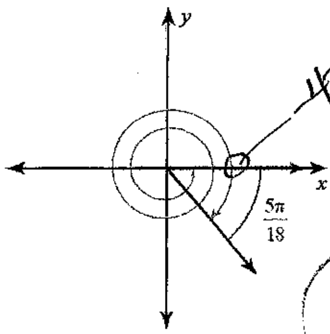


-3π or $-\frac{36\pi}{12}$

$RA = \boxed{\frac{\pi}{12}}$

Find the exact measure of each angle in radians. Non-calculator.

18)

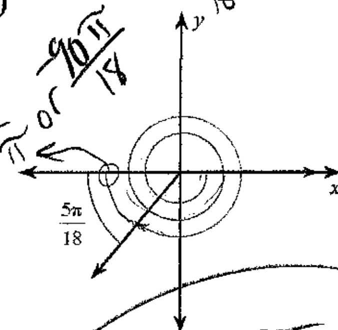


-4π or $-\frac{72\pi}{18}$

$-\frac{5\pi}{18}$

$\theta = -\frac{77\pi}{18}$

19)

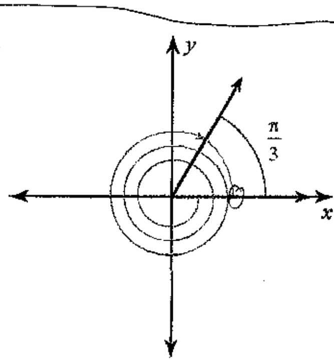


-5π or $-\frac{90\pi}{18}$

$+\frac{5\pi}{18}$

$\theta = -\frac{85\pi}{18}$

20)



-6π or $-\frac{18\pi}{3}$

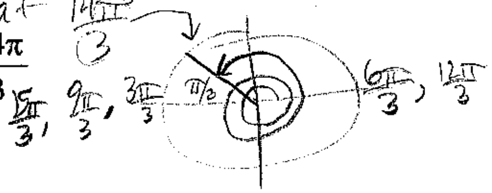
$+\frac{\pi}{3}$

$\boxed{\theta = -\frac{17\pi}{3}}$

Find the exact value of each trigonometric function. Non-Calculator.

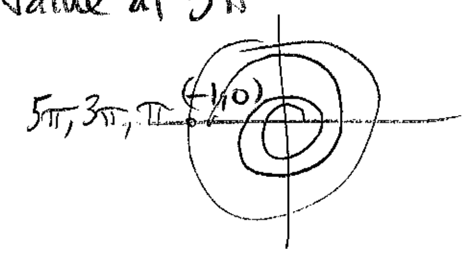
y value at $\frac{14\pi}{3}$

21) $\sin \frac{14\pi}{3}$



$$\boxed{= \frac{\sqrt{3}}{2}}$$

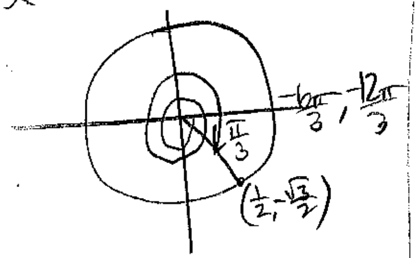
22) $\cot 5\pi$ $\quad \frac{x}{y}$ value at 5π



$$= \frac{-1}{0}$$

$$\boxed{\text{Undefined!}}$$

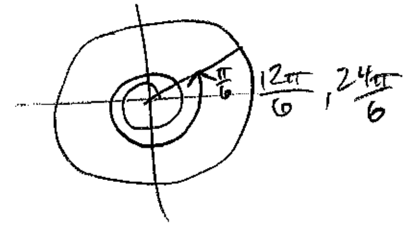
23) $\tan -\frac{13\pi}{3}$ $\quad \frac{y}{x}$ value at $-\frac{13\pi}{3}$



$$= \frac{-\frac{\sqrt{3}}{2}}{\frac{1}{2}}$$

$$\boxed{= -\sqrt{3}}$$

24) $\cos \frac{25\pi}{6}$ $\quad$ x value at $\frac{25\pi}{6}$

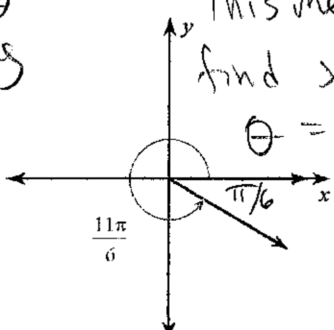


$$\boxed{= \frac{\sqrt{3}}{2}}$$

25) $\cos \theta$

This means find x when $\theta = \frac{11\pi}{6}$

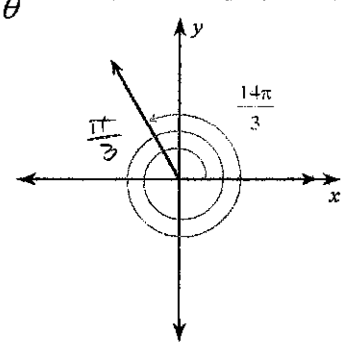
Not solving for θ. θ is given in the picture!



$$\boxed{= \frac{\sqrt{3}}{2}}$$

26) $\csc \theta$

This means find $\frac{1}{\sin\theta}$ or $\frac{1}{y}$ when $\theta = \frac{14\pi}{3}$



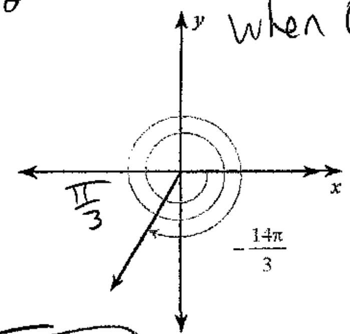
$$= \frac{1}{\frac{\sqrt{3}}{2}}$$

$$= \frac{2}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$$

$$\boxed{= \frac{2\sqrt{3}}{3}}$$

27) $\sin\theta$

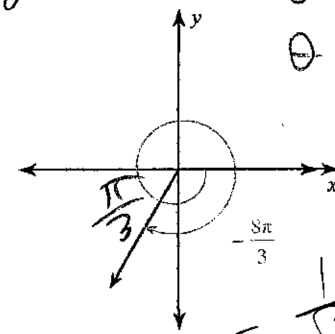
This means y value when $\theta = -\frac{14\pi}{3}$



$$\boxed{= -\frac{\sqrt{3}}{2}}$$

28) $\csc\theta$

This means $\frac{1}{\sin\theta}$ or $\frac{1}{y}$ when $\theta = -\frac{8\pi}{3}$



$$= \frac{1}{-\frac{\sqrt{3}}{2}}$$

$$= -\frac{2}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$$

$$\boxed{= -\frac{2\sqrt{3}}{3}}$$

*Not Unit circles, so find the hyp. when needed.

Use the given point on the terminal side of angle θ to find the value of the trigonometric function indicated. Hint: sketch a right triangle and find the length of the legs and hypotenuse. Use right triangle trigonometry. Calculator portion. SOHCAHTOA

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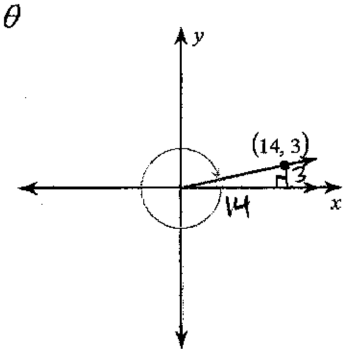
est, but
not #'s
32 & 33.

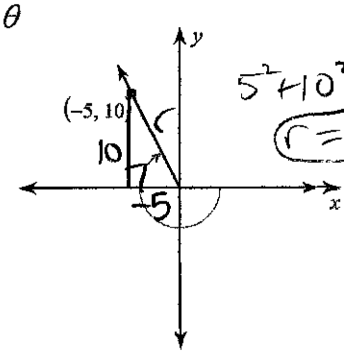
29) $\cot\theta$



$$*\cot\theta = \frac{x}{y}*$$

$$\boxed{\cot\theta = \frac{14}{3}}$$

30) $\cos\theta$



$5^2 + 10^2 = r^2 \rightarrow r = \sqrt{125}$

$r = 5\sqrt{5}$

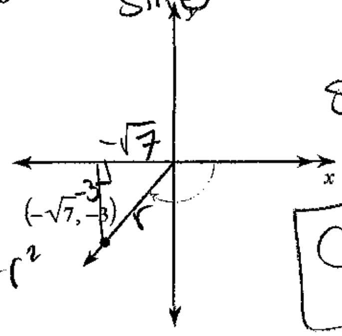
$$\cos\theta = \frac{x}{r}$$ $\frac{\text{adj leg}}{\text{hyp.}}$

$$\cos\theta = -\frac{5}{5\sqrt{5}}$$

$$\cos\theta = -\frac{1}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}}$$

$$\boxed{\cos\theta = -\frac{\sqrt{5}}{5}}$$

31) $\csc\theta = \frac{1}{\sin\theta}$ $\sin\theta = \frac{y}{r}$ ← hyp.



$\sin\theta = -\frac{3}{4}$

$(-\sqrt{7})^2 + (-3)^2 = r^2$

$r = 4$

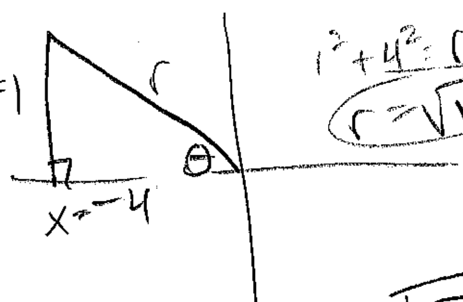
$\boxed{\csc\theta = -\frac{4}{3}}$

*Not on this test!

Find the exact values of the five trigonometric ratios not given. Draw a right triangle to help visualize the length of the legs or hypotenuse. Calculator portion.

QII or QIV QI or QII

32) $\tan\theta = -\frac{1}{4}$ and $\sin\theta > 0$



$1^2 + 4^2 = r^2$

$r = \sqrt{17}$

$\sin\theta = \frac{1}{\sqrt{17}} \cdot \frac{\sqrt{17}}{\sqrt{17}} = \boxed{\frac{\sqrt{17}}{17}}$

$\cos\theta = \frac{-4}{\sqrt{17}} \cdot \frac{\sqrt{17}}{\sqrt{17}} = \boxed{\frac{-4\sqrt{17}}{17}}$

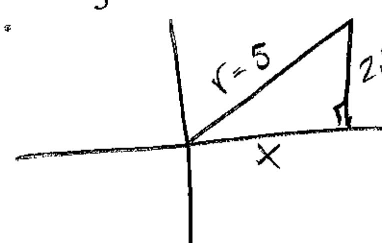
$\sec\theta = \frac{1}{\cos\theta} = \boxed{\frac{\sqrt{17}}{-4}}$

$\csc\theta = \frac{1}{\sin\theta} = \frac{\sqrt{17}}{1}$

$\cot\theta = \frac{-4}{1}$

QI, ~~QII~~ QI, QIV

33) $\sin\theta = \frac{2\sqrt{5}}{5}$ and $\cos\theta > 0$



$x^2 + (2\sqrt{5})^2 = 5^2$

$x = \sqrt{5}$

$\cos\theta = \frac{\sqrt{5}}{5}$

$\tan\theta = \frac{2\sqrt{5}}{\sqrt{5}} = 2$

$\cot\theta = \frac{1}{\tan\theta} = \frac{1}{2}$

$\sec\theta = \frac{1}{\cos\theta} = \frac{5}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \sqrt{5}$

$\csc\theta = \frac{1}{\sin\theta} = \frac{5}{2\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{\sqrt{5}}{2}$

θ must be in radians!

Solve each equation for $\boxed{0 \le \theta < 2\pi.}$ Non-Calculator.

34) $4 + \sin\theta = 5$

$\sin\theta = 1$

What θ has a y-value of 1?

$\boxed{\theta = \frac{\pi}{2}}$

35) $\frac{-12\csc\theta}{-12} = \frac{-8\sqrt{3}}{-12}$

$\csc\theta = \frac{2\sqrt{3}}{3}$

$\frac{1}{\sin\theta} = \frac{2\sqrt{3}}{3}$

$\sin\theta = \frac{3}{2\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$

$\sin\theta = \frac{\sqrt{3}}{2}$ So what θ has a y value of $\frac{\sqrt{3}}{2}$?

$\boxed{\theta = \frac{\pi}{3} \quad \theta = \frac{2\pi}{3}}$

36) $\frac{0}{4} = \frac{4\cot\theta}{4}$

$\cot\theta = 0$

So what θ has $\frac{x}{y} = 0$?

$= \frac{0}{1}$ and $\frac{x}{y} = \frac{0}{-1}$

$\boxed{\theta = \frac{\pi}{2},\ \theta = \frac{3\pi}{2}}$

37) $-3 + \cos\theta = \frac{-9 + 2\sqrt{3}}{3} + 3$
$+3$

$\cos\theta = \frac{-9 + 2\sqrt{3}}{3} + \frac{9}{3}$

$\cos\theta = \frac{-9 + 2\sqrt{3} + 9}{3}$

$\cos\theta = \frac{2\sqrt{3}}{3}$

No solution!

$\frac{2\sqrt{3}}{3}$ is an exact value for secant & cosecant.

38) $\frac{4}{-8} = \frac{-8\sec\theta}{-8}$

$\sec\theta = -\frac{1}{2}$

No solution!

$-\frac{1}{2}$ is an exact value for sine & cosine!

39) $-\frac{\sqrt{3}}{3} = \frac{1}{3} \cdot \tan\theta$ or multiply by 3
$\frac{1}{3}$

$3 \cdot -\frac{\sqrt{3}}{3} = \tan\theta$

$\tan\theta = -\sqrt{3}$

Negative in QII + QIV

$\pi/3$ M

$\boxed{\theta = \frac{2\pi}{3} \quad \theta = \frac{5\pi}{3}}$

* So what θ has $\frac{y}{x} = -\sqrt{3}$

Try $\frac{y}{x} = \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}}$ or $\frac{y}{x} = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}}$

$\sqrt{3}$ $\qquad \frac{\sqrt{3}}{3}$

Referenced to $(\frac{1}{2}, \frac{\sqrt{3}}{2})$ $\pi/3$

When $\frac{y}{x} = \sqrt{3}$

40) $\frac{3\sqrt{3}}{9} = \frac{9\cot\theta}{9}$

$\cot\theta = \frac{\sqrt{3}}{3}$ $\quad \frac{x}{y} = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}}$

$\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right) \rightarrow$ RA of $\frac{\pi}{3}$

$\frac{x}{y}$ is positive in QI + QIII

$\boxed{\theta = \frac{\pi}{3}, \theta = \frac{4\pi}{3}}$

$\frac{3}{2} \cdot$ 41) $\frac{2}{3} \cdot \cos\theta = -\frac{\sqrt{3}}{3} \cdot \frac{3}{2}$

$\cos\theta = -\frac{\sqrt{3}}{2}$

$\boxed{\theta = \frac{5\pi}{6}, \theta = \frac{7\pi}{6}}$

$+\frac{15}{3}$ 42) $-\frac{14}{3} = -5 - \frac{2}{3} \cdot \sin\theta$
$\quad +5$

$-\frac{3}{2} \cdot -\frac{1}{3} = -\frac{2}{3}\sin\theta \cdot -\frac{3}{2}$

$\sin\theta = -\frac{1}{2}$

$\boxed{\theta = \frac{7\pi}{6}, \theta = \frac{11\pi}{6}}$

43) $5 - 3\tan\theta = 5$
$-5 \quad\quad -5$

$-3\tan\theta = 0$

$\tan\theta = 0$

$\frac{y}{x} = 0$ when $\frac{0}{1}$ and $\frac{0}{-1}$

$(1,0)$ $(-1,0)$

$\boxed{\theta = 0, \theta = \pi}$

Not 2π b/c domain is $0 \leq \theta < 2\pi$

less than, not equal to!