HPC - A

Name KEY (Not reduced √)

99 problems, but dividing ain't one

Hour____

Factor completely then find all complex solutions where $f(x) = 0$.

1) $f(x) = 686x^3 - 54$

$f(x) = 2(7x-3)(49x^2+21x+9)$

$x = \frac{3}{7}$ $\quad x = \frac{-21 \pm i\sqrt{1323}}{98}$

2) $f(x) = 32 + 1372x^3$

$f(x) = 4(7x+2)(49x^2-14x+4)$

$x = -\frac{2}{7}$ $\quad x = \frac{14 \pm i\sqrt{588}}{98}$

3) $500x^6 - 108x^3$

$f(x) = 4x^3(5x-3)(25x^2+15x+9)$

$x = 0, x = \frac{3}{5}$ $\quad x = \frac{-15 \pm i\sqrt{675}}{50}$

4) $f(x) = -864 + 500x^3$

$f(x) = 4(5x+6)(25x^2+30x+36)$

$x = -\frac{6}{5}$ $\quad x = \frac{-30 \pm i\sqrt{2700}}{50}$

5) ~~$f(x) = 2x^5 + 6x^4 + 25x^3 + 75x^2 + 72x + 216$; $f(-3) = 0$~~

$f(x) = (x^2+8)(2x^2+9)(x+3)$

$x = -3$ | $x^2+8=0$ | $2x^2+9=0$

$x = \pm i\sqrt{8}$ | $x = \pm i\sqrt{\frac{9}{2}}$

6) $f(x) = 4x^6 - 2x^5 - 24x^4 - 3x^3 - 43x^2 - x - 15$; $f(3) = 0$ and $f\left(-\frac{5}{2}\right) = 0$

$f(x) = (2x+5)(x^2+1)(2x^2+1)(x-3)$

$x = -\frac{5}{2}$ | $x^2+1=0$ | $2x^2+1=0$

$x = 3$ | $x = \pm i$ | $x = \pm i\sqrt{\frac{1}{2}}$

7) $f(x) = -5x^5 - 15x^4 + 18x^3 + 54x^2 + 8x + 24$; $f(-2) = 0$ and $f(-3) = 0$

$f(x) = -(5x^2+2)(x-2)(x+2)(x+3)$

$x = -2$

$x = 2$

$x = -3$

$5x^2 + 2 = 0$

$x = \pm i\sqrt{\frac{2}{5}}$

8) $f(x) = -x^6 - 10x^5 - 11x^4 + 140x^3 + 301x^2 - 490x - 1225$; $f(-5) = 0$ where a bounce occurs at $x = -5$. This means we have two identical factors.

$f(x) = -(x+5)^2(x^2-7)^2$

$x = -5$

$x = \pm\sqrt{7}$

No imaginary solutions here.

Answers to 99 problems, but dividing ain't one

1) Factors to: $f(x) = 2(7x-3)(49x^2+21x+9)$

   Zeros: $\{\frac{3}{7}, \frac{-3+3i\sqrt{3}}{14}, \frac{-3-3i\sqrt{3}}{14}\}$

2) Factors to: $f(x) = 4(7x+2)(49x^2-14x+4)$

   Zeros: $\{-\frac{2}{7}, \frac{1+i\sqrt{3}}{7}, \frac{1-i\sqrt{3}}{7}\}$

3) Factors to: $f(x) = 4x^3(5x-3)(25x^2+15x+9)$

   Zeros: $\{0, \pm\sqrt{\frac{3}{5}}, \frac{-3+3i\sqrt{3}}{10}, \frac{-3-3i\sqrt{3}}{10}\}$

4) Factors to: $f(x) = 4(5x-6)(25x^2+30x+36)$

   Zeros: $\{\frac{6}{5}, \frac{-3+3i\sqrt{3}}{5}, \frac{-3-3i\sqrt{3}}{5}\}$

5) Factors to: $f(x) = (x^2+8)(2x^2+9)(x+3)$

   Zeros: $\{2i\sqrt{2}, -2i\sqrt{2}, \frac{3i\sqrt{2}}{2}, -\frac{3i\sqrt{2}}{2}, -3\}$

6) Factors to: $f(x) = (2x+5)(x^2+1)(2x^2+1)(x-3)$

   Zeros: $\{-\frac{5}{2}, i, -i, \frac{i\sqrt{2}}{2}, -\frac{i\sqrt{2}}{2}, 3\}$

7) Factors to: $f(x) = -(5x^2+2)(x-2)(x+2)(x+3)$

   Zeros: $\{\frac{i\sqrt{10}}{5}, -\frac{i\sqrt{10}}{5}, 2, -2, -3\}$

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