Precalculus Functions Name KEY

1.8 Unique Compositions

Perform a composition and simplify given the two functions (either $f(g(x))$ or $g(f(x))$).

1) $g(x) = (x-1)^3 + 1$
$f(x) = \sqrt[3]{x-1} + 1$

$$f(g(x)) = \sqrt[3]{g(x) - 1} + 1$$
$$= \sqrt[3]{(x-1)^3 + 1 - 1} + 1$$
$$= \sqrt[3]{(x-1)^3} + 1$$
$$= (x-1) + 1$$
$$\boxed{= x}$$

2) $g(x) = 1 + (x+1)^3$
$f(x) = \sqrt[3]{x-1} - 1$

$$g(f(x)) = 1 + (f(x) + 1)^3$$
$$= 1 + (\sqrt[3]{x-1} - 1 + 1)^3$$
$$= 1 + (\sqrt[3]{x-1})^3$$
$$= 1 + x - 1$$
$$\boxed{= x}$$

3) $f(x) = 2(x+2)^5$
$g(x) = \dfrac{-4 + \sqrt[5]{16x}}{2}$

$$f(g(x)) = 2(g(x) + 2)^5$$
$$= 2\left(\frac{-4 + \sqrt[5]{16x}}{2} + \frac{2 \cdot 2}{2}\right)^5$$
$$= 2\left(\frac{\sqrt[5]{16x}}{2}\right)^5$$
$$= 2\left(\frac{16x}{32}\right)$$
$$= \frac{32x}{32}$$
$$\boxed{= x}$$

4) $g(x) = \dfrac{-20 + 6x}{5}$
$f(x) = \dfrac{5x + 20}{6}$

$$f(g(x)) = \frac{5g(x) + 20}{6}$$
$$= \frac{5\left(\frac{-20 + 6x}{5}\right) + 20}{6}$$
$$= \frac{-20 + 6x + 20}{6}$$
$$= \frac{6x}{6}$$
$$\boxed{= x}$$

Find the inverse of each function. If you're unsure of correctness, check your work using the composition method.

5) $f(x) = \frac{4}{x} + 3$

$y = \frac{4}{x} + 3$

$x = \frac{4}{y} + 3$

$-3 \quad -3$

$(x-3) = \frac{4}{y} \cdot y$

$\frac{y(x-3)}{(x-3)} = \frac{4}{(x-3)}$

$\boxed{f^{-1}(x) = \frac{4}{x-3}}$

6) $f(x) = -\frac{2}{x-2} - 2$

$y = \frac{-2}{x-2} - 2$

$x = \frac{-2}{y-2} - 2$

$+2 \quad +2$

$(y-2)(x+2) = \frac{-2}{\cancel{y-2}}\cancel{(y-2)}$

$\frac{(y-2)\cancel{(x+2)}}{\cancel{(x+2)}} = \frac{-2}{x+2}$

$y - 2 = \frac{-2}{x+2} + 2$

$+2$

$\boxed{f^{-1}(x) = \frac{-2}{x+2} + 2}$

7) $h(x) = \sqrt[5]{x-1} - 1$

$y = \sqrt[5]{x-1} - 1$

$x = \sqrt[5]{y-1} - 1$

$+1$

$(x+1)^5 = (\sqrt[5]{y-1})^5$

$(x+1)^5 = y - 1$

$+1 \quad +1$

$\boxed{h^{-1}(x) = (x+1)^5 + 1}$

8) $g(x) = 3 + x^5$

$y = 3 + x^5$

$x = 3 - y^5$

$-3 \quad -3$

$\frac{x-3}{-1} = \frac{-y^5}{-1}$

$\sqrt[5]{-x+3} = \sqrt[5]{y^5}$

$\boxed{g^{-1}(x) = \sqrt[5]{-x+3}}$

9) $g(x) = -2x^5$

$y = -2x^5$

$\frac{x}{-2} = \frac{-2y^5}{-2}$

$\sqrt[5]{\frac{x}{-2}} = \sqrt[5]{y^5}$

$\boxed{g^{-1}(x) = \sqrt[5]{\frac{x}{-2}}}$

10) $h(x) = \sqrt[3]{\frac{x-1}{2}}$

$y = \sqrt[3]{\frac{x-1}{2}}$

$(x)^3 = \left(\sqrt[3]{\frac{y-1}{2}}\right)^3$

$2x^3 = \frac{y-1}{\cancel{2}} \cdot \cancel{2}$

$2x^3 = y - 1$
$+1 \quad +1$

$\boxed{h^{-1}(x) = 2x^3 + 1}$

CHALLENGE! Find the inverse of each function. Check your work using the composition method.

11) $f(x) = \frac{x+2}{3x+5}$

$y = \frac{x+2}{3x+5}$

$(3y+5)x = \frac{y+2}{\cancel{3y+5}} \cancel{(3y+5)}$

$x(3y+5) = y+2$

$3xy + 5x = y + 2 - 5x$
$-y \quad -5x \quad -y$

$3xy - y = 2 - 5x$

$\frac{y(3x-1)}{3x-1} = \frac{2-5x}{3x-1}$

$\boxed{f^{-1}(x) = \frac{2-5x}{3x-1}}$

12) $g(x) = \frac{7-x}{5x-2}$

$y = \frac{7-x}{5x-2}$

$(5y-2)\,x = \frac{7-y}{\cancel{5y-2}} \cancel{(5y-2)}$

$x(5y-2) = 7 - y$

$5xy - 2x = 7 - y$
$+y \quad +2x \quad +2x \quad +y$

$5xy + y = 2x + 7$

$\frac{y(5x+1)}{5x+1} = \frac{2x+7}{5x+1}$

$\boxed{g^{-1}(x) = \frac{2x+7}{5x+1}}$

State if the given functions are inverses. Use the composition method (either $f(g(x))$ or $g(f(x))$).

13) $h(x) = 6x^3 + 3$

$f(x) = \frac{\sqrt[3]{x-3}}{6}$

$f(h(x)) = \frac{\sqrt[3]{h(x)-3}}{6}$

$= \frac{\sqrt[3]{6x^3+3-3}}{6}$

$= \frac{\sqrt[3]{6x^3}}{6}$

$= x\frac{\sqrt[3]{6}}{6}$

Do not divide out, so not inverses!

14) $g(x) = -2(x+3)^3$

$f(x) = \frac{-6-\sqrt[3]{4x}}{2}$

$f(g(x)) = \frac{-6-\sqrt[3]{4g(x)}}{2}$

$= \frac{-6-\sqrt[3]{4(-2(x+3)^3)}}{2}$

$= \frac{-6-\sqrt[3]{-8(x+3)^3}}{2}$

$= \frac{-6-(-2)(x+3)}{2} = \frac{-6+2x+6}{2}$

$= \frac{2x}{2} = \boxed{x}$

Yes, inverses!

15) $g(x) = -2x^5 + 1$

$f(x) = \sqrt[5]{\frac{-x+1}{2}}$

$g(f(x)) = -2(f(x))^5 + 1$

$= -2\left(\sqrt[5]{\frac{-x+1}{2}}\right)^5 + 1$

$= -2\left(\frac{-x+1}{2}\right) + 1$

$= x - 1 + 1$

$\boxed{= x}$ Yes, inverses!

16) $f(x) = \frac{2x-10}{3}$

$h(x) = -\frac{5}{6}x - \frac{10}{3}$

$h(f(x)) = -\frac{5}{6}(f(x)) - \frac{10}{3}$

$= -\frac{5}{6}\left(\frac{2x-10}{3}\right) - \frac{10}{3}$

$= \frac{-10x+50}{18} - \frac{10}{3}\cdot\frac{6}{6}$

$= \frac{-10x+50}{18} - \frac{60}{18}$

$= \frac{-10x-10}{18}$

Does not reduce to x, so not inverses!

17) Assume two functions are inverses of each other. Given any function and its inverse, find $f(f(42))$. Explain your reasoning.

42. Inverse functions "undo" each other w/ opposite operations. The initial input is unchanged by the end.