Review - Rational Functions

Simplify each expression.

1) $\frac{x+6y}{15xy^2} - \frac{x+3y}{15xy^2} = \frac{3y}{15xy^2}$

$\boxed{= \frac{1}{5xy}}$

2) $\frac{(7p-3)}{(7p-3)}\frac{2}{7} - \frac{6}{7p-3} \cdot \frac{7}{7}$

$\frac{14p-6-42}{7(7p-3)} = \boxed{\frac{14p-48}{7(7p-3)}}$ ✓

or

$\frac{2(7p-24)}{7(7p-3)}$

Distribute/multiply by 2!

3) $\frac{(n+5)(n-1)}{(n+5)(n-1)}2 + \frac{n+3}{n^2+4n-5}$

$(n+5)(n-1)$

$= \frac{2n^2+8n-10+n+3}{(n+5)(n-1)}$

$\boxed{= \frac{2n^2+9n-7}{(n+5)(n-1)}}$

Dist. by 3

4) $\frac{8}{8}\frac{8b}{2b^2+b-3} + \frac{3}{8}\frac{(2b+3)(b-1)}{(2b+3)(b-1)}$

$(2b+3)(b-1)$

$\frac{64b+6b^2+3b-9}{8(2b+3)(b-1)} = \boxed{\frac{6b^2+67b-9}{8(2b+3)(b-1)}}$

5) $\frac{20}{5} \cdot \frac{8x}{18} = 4 \cdot \frac{4x}{9}$

$\boxed{= \frac{16x}{9}}$

6) $\frac{10}{19} \cdot \frac{4n^2}{13} = \boxed{\frac{40n^2}{247}}$

7) $\frac{8a^2}{28a+8} \div \frac{8}{28a+8}$

$$\frac{\cancel{8}a^2}{\cancel{4(7a+2)}} \cdot \frac{\cancel{4(7a+2)}}{\cancel{8}}$$

$$\boxed{= a^2}$$

8) $\frac{2v-8}{2} \div \frac{v^2+v-20}{v+7}$

$$\frac{\cancel{2}\cancel{(v-4)}}{\cancel{2}} \cdot \frac{v+7}{(v+5)\cancel{(v-4)}}$$

$$= \boxed{\frac{v+7}{v+5}}$$

I factored this quickly because I noticed a $(7n+6)$ in the denominator.

9) $\frac{7n^2-64n-60}{14n+12} \cdot \frac{10n-4}{5n^2+48n-20}$

$$\frac{\cancel{(7n+6)}(n-10)}{\cancel{2}\cancel{(7n+6)}} \cdot \frac{\cancel{2}\cancel{(5n-2)}}{\cancel{(5n-2)}(n+10)}$$

$$\boxed{\frac{n-10}{n+10}}$$

I took a guess that one of my factors in the numerator was $(7n+6)$ and it worked!

Same idea here

10) $\frac{x+1}{20x-40} \cdot \frac{20x-40}{24x}$

$$\frac{x+1}{\cancel{20(x-2)}} \cdot \frac{\cancel{20(x-2)}}{24x}$$

$$= \boxed{\frac{x+1}{24x}}$$

Doesn't always work, but it did here! Use it here too!

11) $\frac{49k^2-35k-24}{7k+3} \div \frac{49k^2-91k+40}{7k^2+2k-5}$

$$\frac{\cancel{(7n+3)}\cancel{(7n-8)}}{\cancel{7k+3}} \cdot \frac{\cancel{(7k-5)}(k+1)}{\cancel{(7k-5)}\cancel{(7k-8)}}$$

$$= \boxed{k+1}$$

Use other easily found factors to help factor the crazy ones!

Solve each equation. Remember to check for extraneous solutions.

12) $\frac{7}{n^2 + 7n} = \frac{1}{n+7} - \frac{1}{n^2 + 7n}$

$\left[\frac{7}{n(n+7)} = \frac{n}{n}\frac{1}{n+7} - \frac{1}{n(n+7)}\right] n(n+7)$

$7 = n - 1$

$+1 \quad +1$

$\boxed{n = 8}$ ✓

13) $\frac{1}{x^2 + 3x} - \frac{1}{x} = \frac{x-2}{x^3 + 4x^2 + 3x}$

$\left[\frac{(x+1)}{(x+1)}\frac{1}{x(x+3)} - \frac{1(x+1)(x+3)}{x(x+1)(x+3)} = \frac{x-2}{x(x+1)(x+3)}\right] x(x+1)(x+3)$

$x + 1 - x^2 - 4x - 3 = x - 2$

$-x \qquad -1 \quad +2 \quad +2$

$-x^2 - 4x = 0$

$-x(x+4) = 0$

Ext $\boxed{x = 0}$ $\boxed{x = -4}$ ✓

Can't divide by 0

Simplify each expression.

14) $\dfrac{\frac{x-5}{25} + \frac{(x-5)5}{5 \cdot 5}}{\frac{25}{x+5} - \frac{2x-10}{x+5}} = \dfrac{\frac{6x-30}{25}}{\frac{-2x+35}{x+5}}$

$= \frac{6x-30}{25} \cdot \frac{x+5}{-2x+35}$

$\boxed{= \frac{6(x-5)(x+5)}{25(-2x+35)}}$

15) $\dfrac{\frac{4}{4}\frac{4}{2x-5} + \frac{2x-5}{4}\frac{2x-5}{2x-5}}{\frac{2x-5}{2x-5}\frac{2x-3}{x-5} - \frac{4}{2x-5}\frac{x-5}{x-5}} = \dfrac{\frac{16 + 4x^2 - 20x + 25}{4(2x-5)}}{\frac{4x^2 - 16x + 15 - 4x + 20}{(2x-5)(x-5)}}$

$= \frac{4x^2 - 20x + 41}{4(\cancel{2x-5})} \cdot \frac{(\cancel{2x-5})(x-5)}{4x^2 - 20x + 35}$

$\boxed{= \frac{(4x^2 - 20x + 41)(x-5)}{4(4x^2 - 20x + 35)}}$

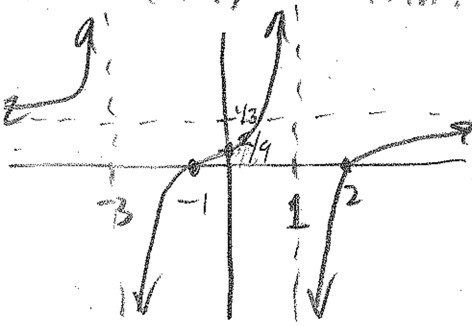
Identify the holes, vertical asymptotes, x-intercepts, and horizontal asymptote of each. Then sketch the graph.

16) $f(x)=\dfrac{x^2-x-2}{3x^2+6x-9}=\dfrac{(x-2)(x+1)}{3(x+3)(x-1)}$

$3(x^2+2x-3)$

x-int: $(2,0)$ & $(-1,0)$ V.A. $x=-3,\ x=1$

y-int: $\left(0,\frac{2}{9}\right)$ H.A. $y=\frac{1}{3}$

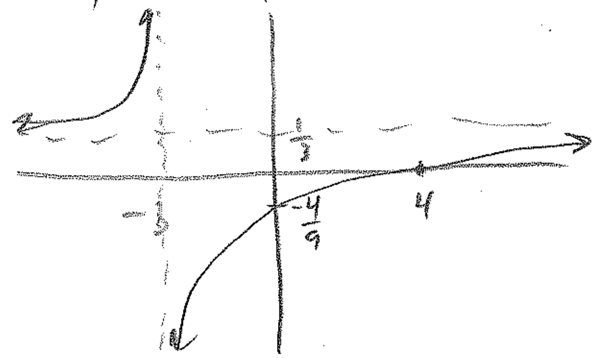


$x\to$	-3^-	-3^+	1^-	1^+
	$x=-3.1$	$x=-2.9$	$x=0.9$	$x=1.1$
$\dfrac{(x-2)(x+1)}{3(x+3)(x-1)}$	$\dfrac{(-)(-)}{(-)(-)}$	$\dfrac{(-)(-)}{(+)(-)}$	$\dfrac{(-)(+)}{(+)(-)}$	$\dfrac{(-)(+)}{(+)(+)}$
$y\to$	$+\infty$	$-\infty$	$+\infty$	$-\infty$

17) $f(x)=\dfrac{x-4}{3x+9}$

x-int: $(4,0)$ V.A. $x=-3$

y-int: $\left(0,-\frac{4}{9}\right)$ H.A. $y=\frac{1}{3}$

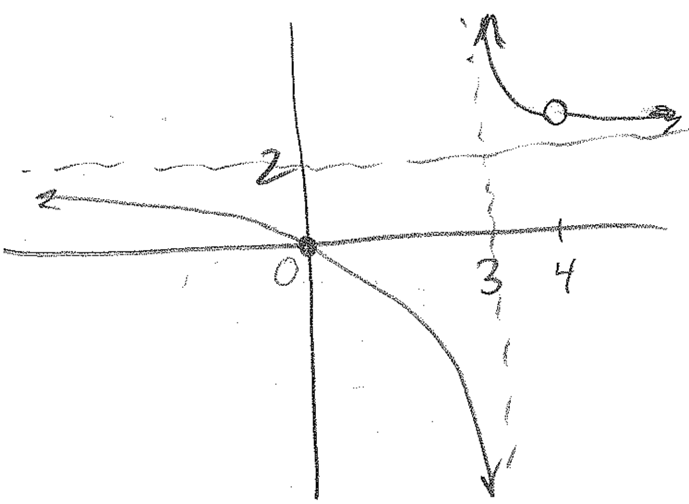


18) $f(x)=\dfrac{2x^2-8x}{x^2-7x+12}=\dfrac{2x\cancel{(x-4)}}{(x-3)\cancel{(x-4)}}=\dfrac{2x}{x-3}$

Hole at $x=4$

x-int: $(0,0)$ V.A. $x=3$

y-int: $(0,0)$ H.A. $y=2$



19) $f(x)=\dfrac{x^2+7x+12}{-2x^2-10x-12}=\dfrac{\cancel{(x+3)}(x+4)}{-2(x+2)\cancel{(x+3)}}$

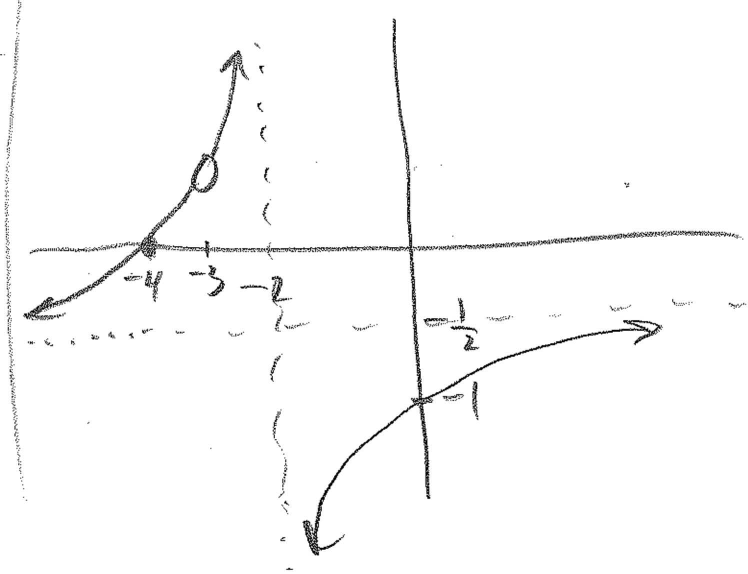
$-2(x^2+5x+6)$

$=\dfrac{x+4}{-2(x+2)}$

Hole at $x=-3$

x-int $(-4,0)$ V.A. $x=-2$

y-int: $(0,-1)$ H.A. $y=-\frac{1}{2}$



Find the domain by factoring completely and then graphing.

20) $\sqrt{x^6-3x^4-25x^2+75}$

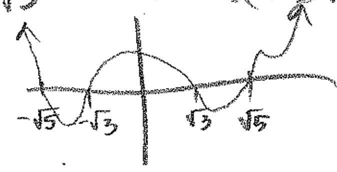
$(x^6-3x^4)-(25x^2-75)\geq 0$

$x^4(x^2-3)-25(x^2-3)\geq 0$

$(x^4-25)(x^2-3)\geq 0$

$(x^2-5)(x^2+5)(x^2-3)\geq 0$

$x=\pm\sqrt{5}$ Not real $x=\pm\sqrt{3}$



D: $(-\infty,-\sqrt{5}]\cup[-\sqrt{3},\sqrt{3}]\cup[\sqrt{5},\infty)$