HPC A Name KEY

1.7 Functions & Domain - Challenge

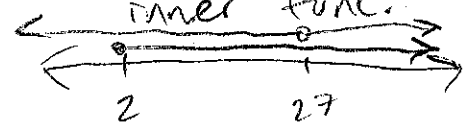
1. Given: $f(x) = \sqrt{x-2}$ *and* $g(x) = \frac{x}{x-5}$, find $g(f(x))$ and its domain.

$g(f(x)) = \frac{f(x)}{f(x)-5} = \boxed{\frac{\sqrt{x-2}}{\sqrt{x-2}-5}}$

$\sqrt{x-2} - 5 \neq 0$

$x \neq 27$

and $[2,\infty)$ for domain of inner func.



$\boxed{D_F: [2,27) \cup (27,\infty)}$

2. Given: $f(x) = \frac{1}{\sqrt{2x-9}}$ *and* $g(x) = x - 4$, find $(f \circ g)(x)$ and its domain.

$f(g(x)) = \frac{1}{\sqrt{2(f(x))-9}} = \frac{1}{\sqrt{2(x-4)-9}} = \boxed{\frac{1}{\sqrt{2x-17}}}$

$2x - 17 > 0$

$x > \frac{17}{2}$

intersect w/ $(-\infty,\infty)$

$\boxed{D_F: \left(\frac{17}{2},\infty\right)}$

3. Given: $f(x) = \sqrt{x-16}$ *and* $g(x) = \sqrt{x-9}$, find $f(g(x))$ *and* the domain.

$f(g(x)) = \sqrt{g(x)-16} = \boxed{\sqrt{\sqrt{x-9}-16}}$

$\sqrt{x-9} - 16 \geq 0$
$+16 \quad +16$

$(\sqrt{x-9})^2 \geq (16)^2$

$x \geq 265$ intersect w/ $[9,\infty)$ inner func.

$\boxed{D_F: [265,\infty)}$

4. Given: $f(x) = \sqrt{x}$, $g(x) = \frac{x}{x-1}$, *and* $h(x) = \sqrt[3]{x}$, find $f(g(h(x)))$ and make an attempt at finding the domain.

$f(g(h(x))) = \sqrt{\frac{\sqrt[3]{x}}{\sqrt[3]{x}-1}}$

$\frac{\sqrt[3]{x}}{\sqrt[3]{x}-1} \geq 0$ and $\sqrt[3]{x} - 1 \neq 0$

$x \neq 1$

Values to check:

$x \rightarrow$	$-\infty, 0$	$0, 1$	$1, \infty$
	$x=-1$	$x=0.5$	$x=2$
$\sqrt[3]{x}$	$(-)$	$(+)$	$(+)$
$\sqrt[3]{x}-1$	$(-)$	$(-)$	$(+)$
$\rightarrow$	$+$	$-$	$+$

$\boxed{D_F: (-\infty,0] \cup (1,\infty)}$

Find the domain for the following functions. Trust the process for finding domain for radical and rational functions.

5. $f(x)=\sqrt{x+8}-\sqrt{9-x}$

D: [-8, ∞)   D: (-∞, 9]

-8   9

$$\boxed{D_F: [-8, 9]}$$

6. $f(x)=\dfrac{\sqrt{5-4x}}{\sqrt{2x-7}}$

$5-4x \geq 0$ $2x-7>0$

$x \leq \frac{5}{4}$ $x > \frac{7}{2}$

$\frac{5}{4}$ $\frac{7}{2}$

Domain does not exist. No real numbers.

7. $f(x)=\dfrac{\sqrt{-5x+12}}{-8x+6}$

$-5x+12 \geq 0$ $-8x+6 \neq 0$

$x \leq \frac{12}{5}$ $x \neq \frac{3}{4}$

$\frac{3}{4}$ $\frac{12}{5}$

$$\boxed{D_F: \left(-\infty, \frac{3}{4}\right) \cup \left(\frac{3}{4}, \frac{12}{5}\right]}$$

8. $f(x)=\dfrac{x^2-9}{\sqrt{3x+16}}$

Numerator's domain is $(-\infty, \infty)$

$3x+16>0$

$x > \frac{-16}{3}$

$$\boxed{D_F: \left(\frac{-16}{3}, \infty\right)}$$

Domain of $3x$ $(-\infty,\infty)$

9. $f(x)=\dfrac{\dfrac{3x}{4x-9}}{\dfrac{4-3x}{18-2x}} = \dfrac{3x}{4x-9}\cdot\dfrac{18-2x}{4-3x}$ which means this function has 3 denominators

$4x-9 \neq 0$ $\quad 18-2x \neq 0$ $\quad 4-3x \neq 0$

$x \neq \frac{9}{4}$ $\quad x \neq 9$ $\quad x \neq \frac{4}{3}$

$$\boxed{D_F: \left(-\infty, \tfrac{4}{3}\right) \cup \left(\tfrac{4}{3}, \tfrac{9}{4}\right) \cup \left(\tfrac{9}{4}, 9\right) \cup (9, \infty)}$$

10. $f(x)=\dfrac{-6x-8}{\sqrt{-4x+5}}$ Numerator has domain $(-\infty,\infty$

$-4x+5 > 0$

$x < \frac{5}{4}$

$$\boxed{D_F: \left(-\infty, \tfrac{5}{4}\right)}$$

11. $f(x)=\dfrac{\dfrac{\sqrt{3-2x}}{x+9}}{\dfrac{x}{1-2x}}$ Similar to # 9, except the numerator has a restricted domain. $\quad \dfrac{\sqrt{3-2x}}{x+9}\cdot\dfrac{1-2x}{x}$

$3-2x \geq 0$ $\quad x+9 \neq 0$ $\quad x \neq 0$ $\quad 1-2x \neq 0$

$x \leq \frac{3}{2}$ $\quad x \neq -9$ $\quad x \neq 0$ $\quad x \neq \frac{1}{2}$

$$\boxed{D_F: (-\infty, -9) \cup (-9, 0) \cup \left(0, \tfrac{1}{2}\right) \cup \left(\tfrac{1}{2}, \tfrac{3}{2}\right]}$$