1.5 Notes – Combining and Composition of Functions

Combining Functions: Here is what you need to know, with a few examples.

When adding, subtracting, multiplying or dividing two or more functions it is important that you first find the domains of each function. The actual adding, subtracting, multiplying, and dividing of these functions will not be the most difficult part. Sometimes the functions are just placed next to each other with the correct operation between them.

In terms of notation:	*Means:*	*Read as:*
$f(x) + g(x) = (f + g)(x)$	Adding functions	*f* of *x* plus *g* of *x* equals *f* plus *g* of *x*
$f(x) - g(x) = (f - g)(x),$		
$f(x)g(x) = (fg)(x),$		
$\frac{f(x)}{g(x)} = \left(\frac{f}{g}\right)(x)\ when\ g(x) \neq 0.$		

When finding the domain for each of these combined functions we have to look at where each individual domains **intersect** as well as the final function. So, the domain of $f(x) \cap g(x)'s$ domain.

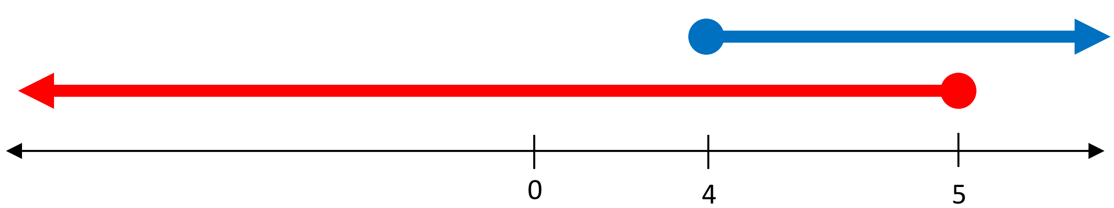
Example: $f(x) = \sqrt{x-4}\ and\ g(x) = \sqrt{5-x}$

First we need to find each individual domain.

$f(x) has\ the\ domain\colon\ [4, \infty)$

$g(x) has\ the\ domain\colon\ (-\infty, 5]$

To find where these overlap, a number line can be very useful!



By looking at this we can see that the number lines overlap in two locations. Hence, the domain for adding, subtracting, and multiplying is $D\colon [4, 5]$. If we divide f(x) by g(x) we have to make sure g(x) ≠ 0. Since g(x) = 0 when x = 5, the domain for $\frac{f(x)}{g(x)}$ is $D\colon [4, 5)$. The only difference is a parentheses on the 5 instead of the square bracket.

Composition of Functions:

The notation for composition of functions and finding the domain is quite different than combining functions. Here is the notation:

In terms of notation:	*Means:*	*Read as:*
$f(g(x)) = (f \circ g)(x)$	Plug *g* into *f* where there is an *x*	*f* of *g* of *x* equals *f* following *g* of *x*
$f(f(x)) = (f \circ f)(x)$	Plug *f* into *f* where there is an *x*	*f* of *f* of *x* equals *f* following *f* of *x*
$g(f(x)) = (g \circ f)(x),$		
$g(g(x)) = (g \circ g)(x)$		

We first find the domain of each function individually. When finding the domain of the composed function we look at where the final function's domain and the inner function's domain intersect. So, the $domain\ of\ f(g(x)) \cap g(x)'s\ domain.$

Example #1:

$$f(x) = \frac{1}{x}\ and\ g(x) = \sqrt{x}$$

We first find the domain of each function.

$f(x) has\ the\ domain: (-\infty, 0) \cup (0, \infty)$, since you can't divide by zero

$g(x) has\ the\ domain\ [0, \infty)$, since you can't square root a negative

Consider $f(g(x)) = \frac{1}{\sqrt{x}}$ where we substituted g(x) into every location of "x" in f(x).

The domain of this function is based on the domain of our inner most function g(x) and our final function f(g(x)).

g(x) was restricted to $[0, \infty)$, however with our new function f(g(x)), we cannot have our denominator equal to 0, so we must make our domain for $f(g(x))\ to\ be\ (0, \infty)$. It sounds easy, right? You could have drawn a number line to find where g(x) and f(g(x))'s domain overlapped/intersected, but for this example it was fairly easy to see that we could just change the square bracket to a parentheses.

Example #2:

$$f(x) = x^2\ and\ g(x) = \sqrt{x - 8}$$

Find $f(g(x))$ and its domain.