HPC - A

Name KEY

Daring to Divide

Use the Remainder Theorem to determine if the given polynomial is divisible by the binomial (with no remainder). If there is no remainder, then divide to find the quotient.

1) $\dfrac{6x^3 - 10x^2 + 4x - 14}{x - 2}$

$x = 2 \quad 6(2)^3 - 10(2)^2 + 4(2) - 14 = 2$

Remainder of 2.

2) $\dfrac{x^3 + 10x^2 + 24x + 66}{x + 8}$

$x = -8 \quad (-8)^3 + 10(-8)^2 + 24(-8) + 66 = 2$

Remainder of 2.

3) $\dfrac{7x^3 - 11x^2 - 3x - 6}{x - 2}$

$x = 2 \quad 7(2)^3 - 11(2)^2 - 3(2) - 6 = 0$

$$\begin{array}{r} 7x^2 + 3x + 3 \\ x-2 \overline{\big)\, 7x^3 - 11x^2 - 3x - 6} \\ -(7x^3 - 14x^2) \\ \hline 3x^2 - 3x \\ -(3x^2 - 6x) \\ \hline 3x - 6 \\ -(3x - 6) \\ \hline 0 \end{array}$$

$\boxed{= 7x^2 + 3x + 3}$

4) $\dfrac{8x^3 - 21x^2 + 7x + 6}{x - 2}$

$x = 2 \quad 8(2)^3 - 21(2)^2 + 7(2) + 6 = 0$

$$\begin{array}{r} 8x^2 - 5x - 3 \\ x-2 \overline{\big)\, 8x^3 - 21x^2 + 7x + 6} \\ -(8x^3 - 16x^2) \\ \hline -5x^2 + 7x \\ -(-5x^2 + 10x) \\ \hline -3x + 6 \\ (-3x + 6) \\ \hline 0 \end{array}$$

$\boxed{= 8x^2 - 5x - 3}$

Completely factor the following polynomials. One rational zero has been given. Find all real solutions where $f(x) = 0$.

5) $f(x) = x^3 + 4x^2 - 3x - 18;\ f(-3) = 0$

$$x^2 + x - 6$$
$$x+3 \,\big)\, x^3 + 4x^2 - 3x - 18$$
$$-(x^3 + 3x^2)$$
$$x^2 - 3x$$
$$-(x^2 + 3x)$$
$$-6x - 18$$
$$-(-6x - 18)$$
$$0$$

$(x+3)(x^2 + x - 6)$

$\boxed{(x+3)(x+3)(x-2) = f(x)}$

$\boxed{x = -3,\ x = 2}$

6) $f(x) = x^4 - x^3 - 17x^2 - 15x;\ f(5) = 0$

$= x(x^3 - x^2 - 17x - 15)$

$$x^2 + 4x + 3$$
$$x-5 \,\big)\, x^3 - x^2 - 17x - 15$$
$$-(x^3 - 5x^2)$$
$$4x^2 - 17x$$
$$-(4x^2 - 20x)$$
$$3x - 15$$
$$-(3x - 15)$$
$$0$$

$x(x-5)(x^2 + 4x + 3)$

$\boxed{x(x-5)(x+1)(x+3) = f(x)}$

$\boxed{x = 0,\ x = 5,\ x = -1,\ x = -3}$

7) $f(x) = x^5 + 5x^4 - 12x^3 - 60x^2 + 36x + 180;\ f(-5) = 0$

$$x^4 - 12x^2 + 36$$
$$x+5 \,\big)\, x^5 + 5x^4 - 12x^3 - 60x^2 + 36x + 180$$
$$-(x^5 + 5x^4)$$
$$0$$
$$-12x^3 - 60x^2$$
$$-(-12x^3 - 60x^2)$$
$$0$$
$$36x + 180$$
$$-(36x + 180)$$
$$0$$

$(x+5)(x^4 - 12x^2 + 36)$

$\boxed{f(x) = (x+5)(x^2 - 6)(x^2 - 6) \text{ or } f(x) = (x+5)(x^2 - 6)^2}$

$\boxed{x = -5,\ x = \pm\sqrt{6}}$ Double roots

8) $f(x) = 9x^5 - 6x^4 - 60x^3 + 40x^2 + 36x - 24;\ f\left(\frac{2}{3}\right) = 0$

Do not use $(x - \frac{2}{3})$ as a factor.

$$3x^4 - 20x^2 + 12$$

$$3x - 2 \,\overline{\big)\, 9x^5 - 6x^4 - 60x^3 + 40x^2 + 36x - 24}$$

$$\underline{-(9x^5 - 6x^4)}$$
$$0$$
$$-60x^3 + 40x^2$$
$$\underline{-(-60x^3 + 40x^2)}$$
$$0$$
$$36x - 24$$
$$\underline{-(36x - 24)}$$
$$0$$

$$(3x-2)(3x^4 - 20x^2 + 12)$$

$$\boxed{f(x) = (3x+2)(3x^2 - 2)(x^2 - 6)}$$

$$\boxed{x = -\frac{2}{3},\ x = \pm\sqrt{\frac{2}{3}},\ x = \pm\sqrt{6}}$$

My preference.

9) $f(x) = -3x^4 - 5x^3 + 24x + 40;\ f\left(-\frac{5}{3}\right) = 0$ $\longrightarrow -1(3x^4 + 5x^3 - 24x - 40)$

$$x^3 - 8$$

$$3x + 5 \,\overline{\big)\, 3x^4 + 5x^3 - 24x - 40}$$

$$\underline{-(3x^4 + 5x^3)}$$
$$0$$
$$-24x - 40$$
$$\underline{-(-24x - 40)}$$
$$0$$

$$-(3x+5)(x^3 - 8)$$

$$\boxed{f(x) = -(3x+5)(x-2)(x^2 + 2x + 4)}$$

$$\boxed{x = -\frac{5}{3},\ x = 2}$$

10) $f(x)=6x^5+30x^4-8x^3-40x^2+2x+10;\ f(-5)=0$

$$f(x)=2(3x^5+15x^4-4x^3-20x^2+x+5)$$

$$\begin{array}{r|l} & 3x^4-4x^2+1 \\ \hline x+5 & 3x^5+15x^4-4x^3-20x^2+x+5 \\ & -(3x^5+15x^4) \\ & \quad 0 \quad -4x^3-20x^2 \\ & \quad -(-4x^3-20x^2) \\ & \quad\quad 0 \quad x+5 \\ & \quad\quad -(x+5) \\ & \quad\quad\quad 0 \end{array}$$

$$2(x+5)(3x^4-4x^2+1)$$

$$2(x+5)(3x^2-1)(x^2-1)$$

$$\boxed{f(x)=2(x+5)(3x^2-1)(x+1)(x-1)}$$

$$\boxed{x=-5,\ x=\pm\sqrt{\tfrac{1}{3}},\ x=-1,\ x=1}$$