HPC – A Name ____________________

Notes: Finding solutions to higher degree polynomials
Not in Factored form

Rational Zeros Theorem: If polynomial $P(x) = a_n x^n + a_{n-1}x^{n-1} \ldots + \; a_1 x + a_0$ has integer coefficients, then every rational zero of P is of the form $\frac{p}{q}$ where p is a factor of the constant coefficient $a_0$and q is a factor of the leading coefficient a_n. If $a_n = 1$, then all rational zeros are integers.

Example: How can we find possible solutions without graphing?

$P(x) = 2x^3 + x^2 - 13x + 6$

We find factors of constant 6 and divide them by factors of leading coefficient 2.

$\frac{Factors\ of\ 6}{Factors\ of\ 2} = \; \pm\frac{1}{2}, \pm 1, \pm\frac{3}{2}, \pm 2, \pm 3, \pm 6$. These were found by listing out all the individual factors for each number and then dividing. Some were repeated and those were left out. We use the symbol ± because we could have two negative numbers as a factor pair.

Now we check to see which ones are actual solutions/roots/zeros by using the next theorem!

Factor Theorem: $P(c) = 0$ if and only if $(x - c)$ is a factor of $P(x)$.

So for the above example we would check each one of our 12 possible solutions to see which ones are truly zeros/roots.

$P(-3), P\left(\frac{1}{2}\right) = 0, P(2) = 0$

This should make sense since we have a cubic; there are 3 possible real solutions! We could now write an appropriate equation in factored form.

Using the “a” value of 2 (from the above equation), we would have $P(x) = (x + 3)(2x - 1)(x - 2)$.

Remainder Theorem: If the polynomial $P(x)$ is divided by $(x - c)$, then the remainder is the value of $P(c)$.

This means if we were to divide the above equation $P(x) = 2x^3 + x^2 - 13x + 6$ by any of the three factors $(x + 3), (2x - 1), or (x - 2)$ then we should get a remainder of 0. The reason based on the Remainder Theorem is; $P(-3), P\left(\frac{1}{2}\right) = 0, P(2) = 0$.

These factors will not always be easy to come by, thus we must introduce long division and synthetic division of polynomials. Two strategies that will help us factor higher degree polynomials!

Fundamental Theorem of Algebra: The fundamental theorem of algebra states that every non-constant single-variable polynomial with complex coefficients has *at least one* complex root. This includes polynomials with real coefficients, since every real number is a complex number with an imaginary part equal to zero.

Examples:

$P(x) = x^4 + 2x^3 - 7x^2 - 8x + 12$

$f(x) = 15x^4 - 55x^3 + 33x^2 - 11x + 6\,; f(3) = 0$