# 8.1.2 What is its measure?

### Interior Angles of Polygons

- **8-17.** On graph paper, graph Δ*ABC* if *A*(3, 0), *B*(2, 7), and *C*(6, 4). <u>8-17 HW eTool</u> (Desmos).
- a. What is the most specific name for this triangle? Prove your answer is correct using both slope and side length.

b. Find  $m \angle A$ . Explain how you found your answer.

**8-18.** The exterior angles of a quadrilateral are labeled *a*, *b*, *c*, and *d* in the diagram below. Find the measures of *a*, *b*, *c*, and *d* and then find the sum of the exterior angles.







**8-19.** Find the area and perimeter of the shape below. Show all work.



**8-20.** Crystal is amazed! She graphed  $\triangle ABC$  using the points A(5, -1), B(3, -7), and C(6, -2). Then she rotated  $\triangle ABC$  90° counterclockwise ( $\circlearrowleft$ ) about the origin to find  $\triangle A'B'C'$ . Meanwhile, her teammate took a different triangle ( $\triangle TUV$ ) and rotated it 90° clockwise ( $\circlearrowright$ ) about the origin to find  $\triangle T'U'V'$ . Amazingly,  $\triangle A'B'C'$  and  $\triangle T'U'V'$  ended up using exactly the same points! Name the coordinates of the vertices of  $\triangle TUV$ .



8-22. Suzette started to set up a proof to show that if  $\overline{BC} / \overline{EF}$ ,  $\overline{AB} / \overline{DE}$  and AF = DC,

then  $BC \cong \overline{EF}$ . Examine her work below. Then complete her missing statements and reasons.



# 8.1.3 What if it is a regular polygon?

8-29. Find the area and perimeter of each shape below. Show all work.





a.

**8-30.** In the figure below, if PQ = SR and PR = SQ, prove that  $\angle P \cong \angle S$ . Write your proof either in a flowchart or in two- column proof form.



- **8-31.** Joey used 10 congruent triangles to create a regular decagon.
- a. What kind of triangles is he using?
- b. Find the three angle measures of one of the triangles. Explain how you know.

c. If the area of each triangle is 14.5 square inches, then what is the area of the regular decagon? Show all work.

- **8-32.** On graph paper, plot *A*(2, 2) and *B*(14, 10).
  - a. If *C* is the midpoint of  $\overline{AB}$ , *D* is the midpoint of  $\overline{AC}$ , and *E* is the midpoint of  $\overline{CD}$ , find the coordinates of *E*.

b. What fraction of the distance from *A* to *B* is A to *E*?



c. Use the ratio from part (b) to find the coordinates of point *E*.

**8-33.** Use the geometric relationships in the diagrams below to solve for *x*.



**8-34.** The arc below is called a **quarter circle** because it is one-fourth of a circle.



a. Copy Region A below onto your paper. If this region is formed using 4 quarter circles, can you find another shape that must have the same area as Region A? Justify your conclusion.



b. Find the area of Region A. Show all work.

### 8-35. Multiple Choice: Which property below can be used to prove that the triangles at right are similar?

- a.  $AA \sim$
- b. SAS  $\sim$
- c. SSS ~
- d. HL ~
- e. None of these



# **8.1.4** Is there another way?

### Regular Polygon Angle Connections

**8-40.** Esteban used a hinged mirror to create an equilateral triangle, as shown in the diagram below. If the area of the shaded region is 11.42 square inches, what is the area of the entire equilateral triangle? Justify your solution.



8-41. A house purchased for \$135,000 has an annual appreciation of 4%.

- a. What is the multiplier?
- b. Write a function of the form  $f(t) = ab^t$  that represents the situation, where t is the time in years after the house was purchased.
- c. At the current rate, what will be the value of the house in 10 years?

**8-42.** Copy each shape below on your paper and state if the shape is convex or non-convex. You may want to compare each figure with the examples provided in problem 8-4.



**8-43.** Find the area of each figure below. Show all work. a.



b.



**8-44.** Find the number of sides in a regular polygon if each interior angle has the following measures.

a. 60° b. 156° c. 90° d. 140°

8-45. Determine if the figures below (not drawn to scale) are similar. Justify your decision.



**8-46.** Below is a scale drawing of the floor plan for Nzinga's dollhouse. The actual dimensions of the dollhouse are 5 times the measurements provided in the floor plan below.

 Use the measurements provided in the diagram to find the area and perimeter of her floor plan.



- b. Draw a similar figure on your paper. Label the sides with the actual measurements of Nzinga's dollhouse. What is the perimeter and area of the floor of her actual dollhouse? Show all work.
- c. Find the ratio of the perimeters of the two figures. What do you notice?
- d. Find the ratio of the areas of the two figures. How does the ratio of the areas seem to be related to the zoom factor?

# 8.1.5 What is the area?

### Finding Areas of Regular Polygons



a. What is the measure of an interior angle of this polygon? Show how you know.

b. How many sides does this polygon have? Show all work.

8-56. Find the area of an equilateral triangle with side length 20 mm. Draw a diagram and show all work.

**8-57.** A hotel in Las Vegas is famous for its large-scale model of the Eiffel Tower. The model, built to scale, is 128 meters tall and 41 meters wide at its base. If the real tower is 324 meters tall, how wide is the base of the real Eiffel Tower?

**8-60.** Solve for *x* in each diagram below.



a.









d.

### 8.2.1 How does the area change?



Area Ratios of Similar Figures

#### 8-71. Examine the shape below.



a. Find the area and perimeter of the shape.

b. On graph paper, enlarge the figure so that the linear scale factor is 3. Find the area and perimeter of the new shape.

c. What is the ratio of the perimeters of both shapes? What is the ratio of the areas?

**8-72.** Sandip noticed that when he looked into a mirror that was lying on the ground 8 feet from him, he could see a clock on the wall. If Sandip's eyes are 64 inches off the ground, and if the mirror is 10 feet from the wall, how high above the floor is the clock? Include a diagram in your solution.

**8-73.** Mr. Singer has a dining table in the shape of a regular hexagon. While he loves this design, he has trouble finding tablecloths to cover it. He has decided to make his own tablecloth!

In order for his tablecloth to drape over each edge, he will add a rectangular piece along each side of the regular hexagon as shown in the diagram below. Using the dimensions given in the diagram, find the total area of the cloth Mr. Singer will need.



**8-76.** If the rectangles below have the same area, find *x*. Is there more than one answer? Show all work.



**8-77. Multiple Choice:** A cable 100 feet long is attached 70 feet up the side of a building. If it is pulled taut (i.e., there is no slack) and staked to the ground as far away from the building as possible, approximately what angle does the cable make with the ground?

a. 39.99°

- b. 44.43°
- c. 45.57°
- d. 12.22

### 8.2.2 How does the area change?

Ratios of Similarity

8-83. Assume Figure A and Figure B, below, are similar.



a. If the ratio of similarity is  $\frac{3}{4}$  then what is the ratio of the perimeters of A and B?

b. If the perimeter of Figure A is p and the linear scale factor is r, what is the perimeter of Figure B?

c. If the area of Figure A is *a* and the linear scale factor is *r*, what is the area of Figure B?

**8-84.** Always a romantic, Marris decided to bake his girlfriend a cookie in the shape of a regular dodecagon (12-gon) for Valentine's Day.

a. If the edge of the dodecagon is 6 cm, what is the area of the top of the cookie?

b. His girlfriend decides to divide the cookie into 12 congruent pieces. After 9 of the pieces have been eaten, what area of the cookie is left?

**8-85.** As her team was building triangles with linguini, Karen asked for help building a triangle with sides 5, 6, and 1. *"I don't think that's possible,"* said her teammate, Kelly.

- a. Why is this triangle not possible?
- b. Change the lengths of one of the sides so that the triangle is possible.

**8-87.** Callie started to prove that given the information in the diagram below, then  $\overline{AB} \cong \overline{CD}$ . Copy her flowchart below on your paper and help her by justifying each statement.



8-88. For each pair of triangles below, decide if the triangles are congruent. If the triangles are congruent:

- State which triangle congruence property proves that the triangles are congruent.
- Write a congruence statement (such as  $\triangle ABC \cong \triangle$  \_\_\_\_\_).



8-89. Multiple Choice: What is the solution to the system of equations below?

 $y = \frac{1}{2}x - 4$ x - 4y = 12

- a. (2,0)
- b. (16, 4)
- c. (-2, -5)
- d. (4, -2)
- e. None of these

8.3.1 What if it has infinitely many sides?





**8-93.** Find the area of the shaded region for the regular pentagon below if the length of each side of the pentagon is 10 units. Assume that point *C* is the center of the pentagon.



**8-94.** For each triangle below, find the value of *x*, if possible. Name the triangle tool that you used. If the triangle cannot exist, explain why.

a.



b.









**8-95.** Find the measure of each interior angle of a regular 30-gon using two different methods.

**8-96.** Examine the diagram below. Assume that  $\overline{AD}$  and  $\overline{BE}$  are line segments, and that  $\overline{BC} \cong \overline{DC}$  and  $\angle A \cong \angle E$ . Prove that  $\overline{AB} \cong \overline{ED}$ . Use the form of proof that you prefer (such as the flowchart or two-column proof format). Be sure to copy the diagram onto your paper and add any appropriate markings.



**8-97.** For each diagram below, write and solve an equation to find *x*.



**8-98.** On graph paper, plot the points *A*(−3, −1) and *B*(6, 11).

- a. Find the midpoint M of  $\overline{AB}$ .
- b. Find the point *P*, on  $\overline{AB}$ , that is  $\frac{2}{3}$  of the way from *A* to *B*.
- c. Find the equation of the line that passes through points *A* and *B*.
- d. Find the distance between points *M* and *P*.



8-99. Multiple Choice: What fraction of the circle below is shaded?



e. None of these

## 8.3.2 What is the relationship?



Area and Circumference of a Circle

**8-105.** A regular hexagon with side length 4 has the same area as a square. What is the length of the side of the square? Explain how you know.

8-106. Use what you know about similar figures to complete the following tasks.

a. Find the area and perimeter of the trapezoid below.



b. Find the area and perimeter of the trapezoid that is similar to this one, but has been reduced by a linear scale factor of  $\frac{1}{3}$ .

8-107. An exterior angle of a regular polygon measures 18°.

a. How many sides does the polygon have?

b. If the length of a side of the polygon is 2 units, what is the area of the polygon?

**8-108.** Find the missing angle(s) in each problem below using the geometric relationships shown in the diagram below. Be sure to write down the conjecture that justifies each calculation. Remember that each part is a separate problem.

a. If  $d = 110^{\circ}$  and  $k = 5x - 20^{\circ}$ , write an equation and solve for x.



b. If  $b = 4x - 11^{\circ}$  and  $n = x + 26^{\circ}$ , write an equation and solve for x. Then find the measure of  $\angle n$ .

8-109. Reynaldo has a stack of blocks on his desk, as shown below.



a. If his stack is 2 blocks wide, 2 blocks long, and 2 blocks tall, how many blocks are in his stack?

- b. What if his stack instead is 3 blocks wide, 3 blocks long, and 2 blocks tall? How many blocks are in this stack?
- c. What if his stack contains 99 blocks, is 3 blocks tall, *x* blocks long, and *y* blocks wide? What could *x* and *y* be?
- **8-110.** Krista is trying to solve for *x* in the triangle below.



- a. What equation would Krista write?
- b. Krista does not have a calculator, but she remembered something funny her friend Juanisha told her. Juanisha's favorite sine ratio is sin 31.5° ≈ 0.522 because 5/22 is Juanisha's birthday! Without a calculator, use Juanisha's favorite ratio to solve your equation from part (a).

**8-111. Multiple Choice:** Which type of quadrilateral below does not necessarily have diagonals that bisect each other?

- a. square
- b. rectangle
- c. rhombus
- d. trapezoid

# **8.3.3** How can I use it?



### Circles in Context

**8-116.** Use what you know about the area and circumference of circles to answer the questions below. Show all work. Leave answers in terms of  $\pi$ .

- a. If the radius of a circle is 14 units, what is its circumference? What is its area?
- b. If a circle has diameter 10 units, what is its circumference? What is its area?
- c. If a circle has circumference  $100\pi$  units, what is its area?
- d. If a circle has circumference C, what is its area in terms of C?

**8-117.** In 2000 a share of Jiffy Stock was worth \$20 and in 2010 it was worth \$50. What was the annual multiplier and the annual percent of increase?

**8-118.** Larry started to set up a proof to show that if  $\overline{AB} \perp \overline{DE}$  and  $\overline{DE}$  is a diameter of *ircle with center at C*, then  $\overline{AF} \cong \overline{FB}$ . Examine his work below. Then complete his missing statements and reasons.



Statements	Reasons
1. $\overline{AB} \perp \overline{DE}$ and is a diameter of <i>circle with center at C</i> .	1.
2. ∠AFC and ∠BFC are right angles.	2.
3. <i>FC</i> = <i>FC</i>	3.
4. <i>AC</i> = <i>BC</i>	4. Definition of a Circle (radii must be equal)
5.	5. HL ≅
$6. \overline{AF} \cong \overline{FB}$	6.

8-119. Match each regular polygon named on the left with a statement about its qualities listed on the right.

- a. regular hexagon (1) Central angle of 36°
- b. regular decagon (2) Exterior angle measure of 90°
- c. equilateral triangle (3) Interior angle measure of 120°
- d. square (4) Exterior angle measure of 120°

**8-120.** Examine the graph of f(x) below. Use the graph to find the following values.



- a. *f*(1)
- b. *f*(0)
- c. x if f(x) = 4
- d. x if f(x) = 0

**8-121.** Examine the relationships that exist in the diagram below. Find the measures of angles *a*, *b*, *c*, and *d*. Remember that you can find the angles in any order, depending on the angle relationships you use.



#### 8-122. Multiple Choice: How many cubes with edge length 1 unit would fit in a cube with edge length 3 units?



**8-123.** The city of Denver wants you to help build a dog park. The design of the park is a rectangle with two semicircular ends. (Note: A semicircle is half of a circle.)



- a. The entire park needs to be covered with grass. If grass is sold by the square foot, how much grass should you order?
- b. The park also needs a fence for its perimeter. A sturdy chain-linked fence costs about \$8 per foot. How much will a fence for the entire park cost?

c. The local design board has rejected the plan because it was too small. "Big dogs need lots of room to run," the president of the board said. Therefore, you need to increase the size of the park with a linear scale factor of 2. What is the area of the new design? What is the perimeter?

**8-125.**  $\overline{BE}$  is the midsegment of  $\triangle ACD$ , shown below.

a. Find the perimeter of  $\triangle ACD$ .



b. If the area of  $\triangle ABE$  is 19 cm<sup>2</sup>, what is the area of  $\triangle ACD$ ?

**8-126.** Christie has tied a string that is 24 cm long into a closed loop, like the one below.

- a. She decided to form an equilateral triangle with her string. What is the area of the triangle?
- b. She then forms a square with the same loop of string. What is the area of the square ? Is it more or less than the equilateral triangle she created in part (a)?
- c. If she forms a regular hexagon with her string, what would be its area? Compare this area with the areas of the square and equilateral triangle from parts (a) and (b).

d. What shape should Christie form to enclose the greatest area?

**8-129.** Multiple Choice: The diagram below is not drawn to scale. If  $\triangle ABC \sim \triangle KLM$ , find KM.

- a. 6
- b. 12
- c. 15
- d. 21
- e. None of these

