HPC - A

Name KEY

Review - Polynomial Functions

Date _____

Identify the domain and range. Use interval notation.

1) $f(x) = -2\left(x - \frac{3}{2}\right)^2 - 1$

D: $(-\infty, \infty)$

R: $(-\infty, -1]$

2) $y = 2\sqrt{-6x - 15}$

$-6x - 15 \geq 0$

$+15 \quad +15$

$\frac{-6x}{-6} \geq \frac{15}{-6}$

$x \leq -\frac{5}{3}$

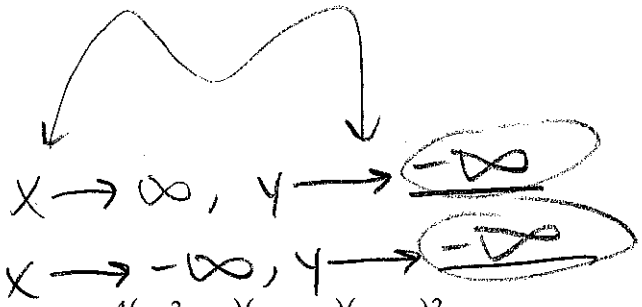
D: $(-\infty, -\frac{5}{3}]$

R: $[0, \infty)$

State the degree of each polynomial and sketch the general shape of the graph. Write the end behavior for each.

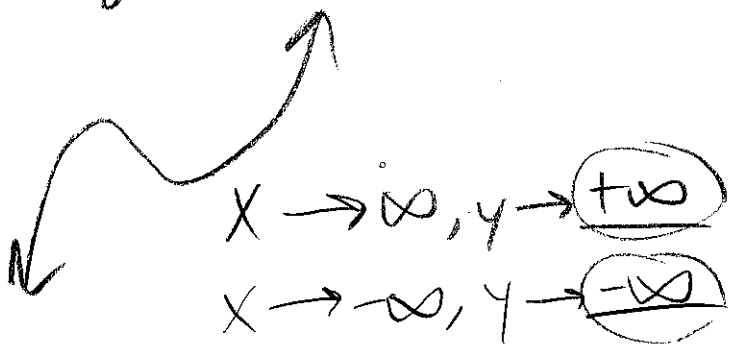
3) $-6x^4 + 9x^3 + x^2 - 15x$

Deg. 4 (-a)



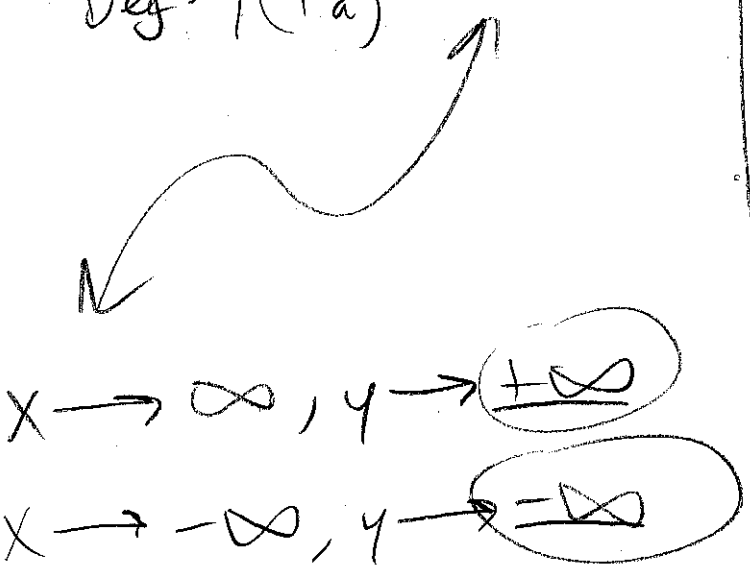
4) $3x^2 - 9x^3 + 15x^5 - 3x$

Deg 5 (+a)



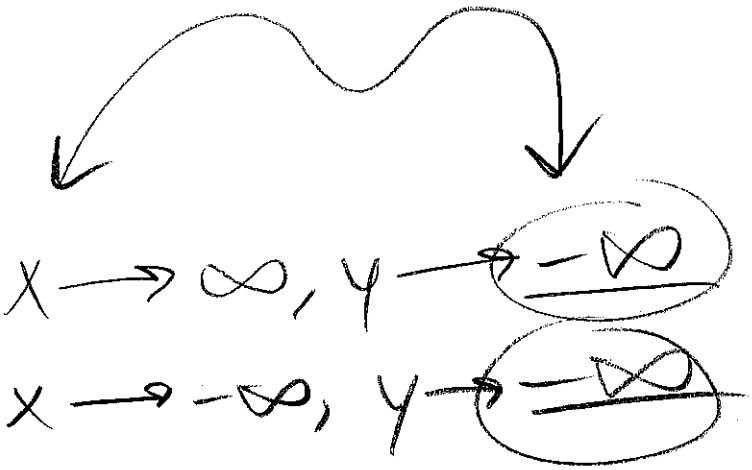
5) $x^4(3x^2 + 2)(4x - 9)(x + 9)^2$

Deg: 9 (+a)



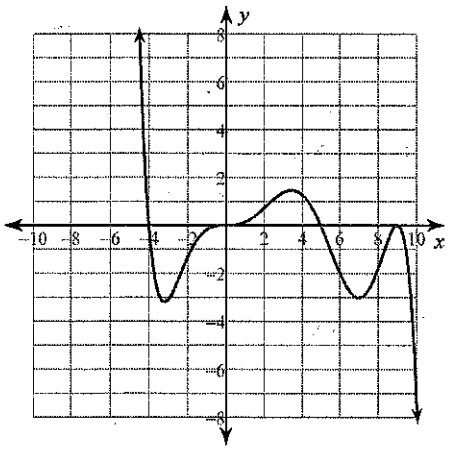
6) $-5x^2(x^2 - 8)^3(x + 5)(x - 4)(x + 10)^2$

Deg: 12 (-a)



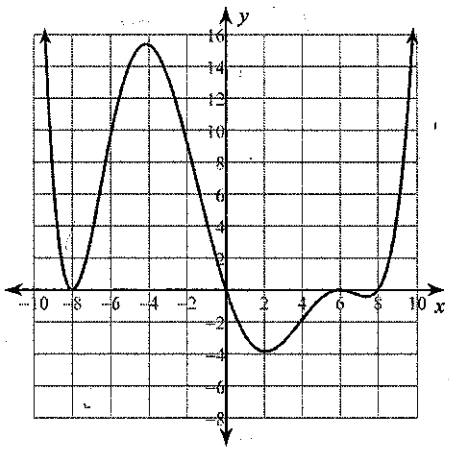
Write an equation for the graphed polynomial.

7)



$f(x) = -x^3(x+4)(x-5)(x-9)^2$

8)

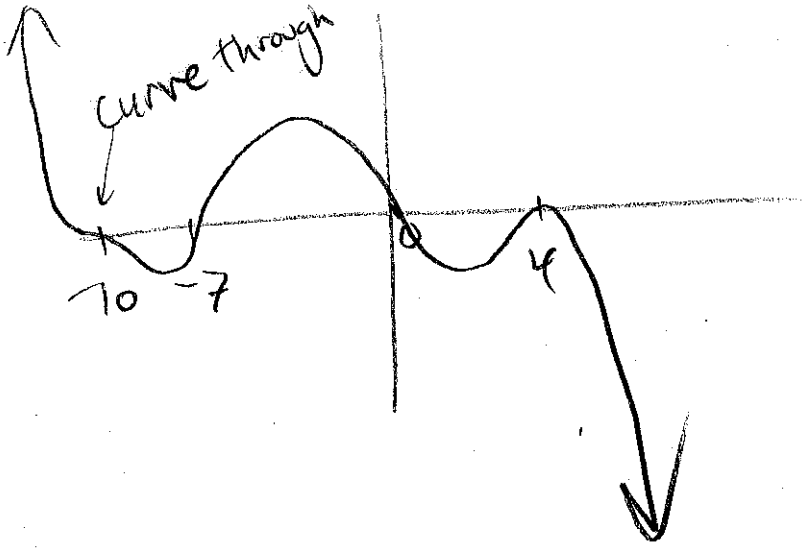


$f(x) = x(x+8)^2(x-6)^2(x-8)$

Sketch a graph of the polynomial. Find all real and complex solutions. Clearly label the x-intercepts (real solutions only) on the graph.

9) $f(x) = -x(x-4)^2(x+7)(x+10)^3$

Deg: 7 (-a)



10) $f(x) = x^4 - 7x^3 + 10x^2 + 14x - 24$; $f(3) = 0$

$$x-3 \overline{)\, x^4 - 7x^3 + 10x^2 + 14x - 24}$$

$x^3 - 4x^2 - 2x + 8$

$-(x^4 - 3x^3)$ ↓

$-4x^3 + 10x^2$

$-(-4x^3 + 12x^2)$ ↓

$-2x^2 + 14x$

$-(-2x^2 + 6x)$ ↓

$8x - 24$

$-(8x - 24)$

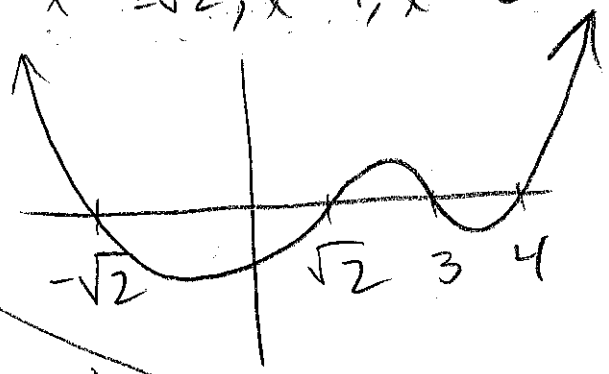
0

$f(x) = (x-3)(x^3 - 4x^2 - 2x + 8)$

-2	$-2x$	$+8$
x^2	x^3	$-4x^2$
	x	-4

$f(x) = (x^2-2)(x-4)(x-3)$ Deg 4 +a

$x = \pm\sqrt{2}, x = 4, x = 3$



*Does not group

11) $f(x) = x^4 + 6x^3 + 4x^2 - 40x$; $f(2) = 0$

$x(x^3 + 6x^2 + 4x - 40)$

$$x-2 \overline{)\, x^3 + 6x^2 + 4x - 40}$$

$x^2 + 8x + 20$

$-(x^3 - 2x^2)$ ↓

$8x^2 + 4x$

$-(8x^2 - 16x)$ ↓

$20x - 40$

$-(20x - 40)$

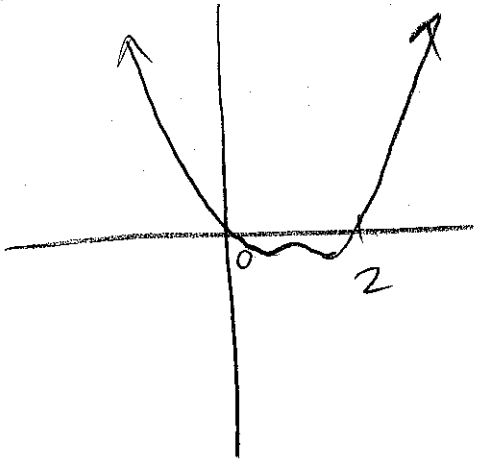
0

$f(x) = x(x-2)(x^2 + 8x + 20)$

Try, but does not factor.

Find x-int w/ $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = 0, x = 2$ Deg 4 (+a)



$x = \dfrac{-8 \pm \sqrt{8^2 - 4(1)(20)}}{2(1)}$

$x = \dfrac{-8 \pm \sqrt{-16}}{2}$

$x = \dfrac{-8 \pm 4i}{2}$

$x = -4 \pm 2i$

Might choose to group

12) $f(x) = -5x^4 - 10x^3 + 40x + 80;\ f(-2) = 0$

$f(x) = -5(x^4 + 2x^3 - 8x - 16)$

$f(x) = -5(x+2)(x^3 - 8)$

$x+2 \overline{)\ x^4 + 2x^3 - 8x - 16}$ → $x^3 - 8$

$-(x^4 + 2x^3)$

0

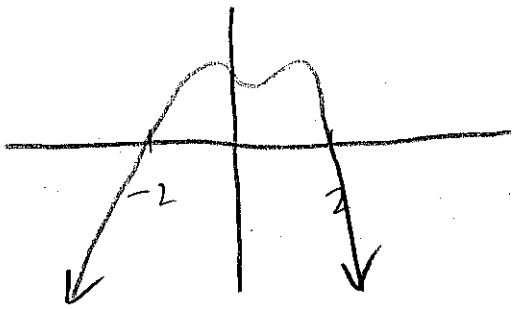
$-8x - 16$

$-(-8x - 16)$

0

$f(x) = -5(x+2)(x-2)(x^2+2x+4)$

$x = -2,\ x = 2$ Deg. 4 (-a)



Minimum

$x = \dfrac{-2 \pm \sqrt{2^2 - 4(1)(4)}}{2(1)}$ → $x = \dfrac{-2 \pm i\sqrt{12}}{2}$

$x = \dfrac{-2 \pm \sqrt{-12}}{2}$

Extra → $x = -1 \pm i\sqrt{3}$

13) $f(x) = 81x^7 - 135x^6 - 57x^4 + 95x^3 - 24x + 40;\ f\left(\frac{5}{3}\right) = 0$

$3x-5 \overline{)\ 81x^7 - 135x^6 - 57x^4 + 95x^3 - 24x + 40}$ → $27x^6 - 19x^3 - 8$

$-(81x^7 - 135x^6)$

0

$-57x^4 + 95x^3$

$-(-57x^4 + 95x^3)$

0

$-24x + 40$

$-(-24x + 40)$

0

$f(x) = (3x-5)(27x^6 - 19x^3 - 8)$

8	$8x^3$	-8
+		
$27x^3$	$27x^6$	$-27x^3$

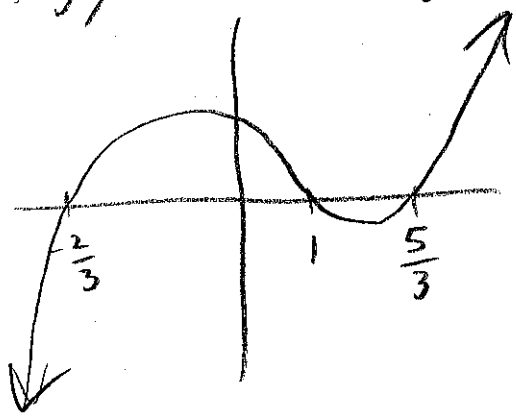
$-216x^6$, $8x^3 - 27x^3$, $-19x^3$

$x^3 - 1$

$f(x) = (3x-5)(27x^3+8)(x^3-1)$

$f(x) = (3x-5)(3x+2)(9x^2-6x+4)(x-1)(x^2+x+1)$

$x = \frac{5}{3}, -\frac{2}{3}, 1$ Deg: 7 (+a)



$x = \dfrac{6 \pm \sqrt{(-6)^2 - 4(9)(4)}}{2(9)}$

$x = \dfrac{6 \pm \sqrt{-108}}{18}$

Minimum

$x = \dfrac{6 \pm i\sqrt{108}}{18}$

$x = \dfrac{1 \pm i\sqrt{3}}{3}$ Extra

$x = \dfrac{-1 \pm \sqrt{1^2 - 4(1)(1)}}{2(1)}$

$x = \dfrac{-1 \pm \sqrt{-3}}{2}$

$x = \dfrac{-1 \pm i\sqrt{3}}{2}$

Find the domain. Use interval notation.

14) $f(x) = \sqrt{-63x^3 - 42x^2 + 168x + 112}$

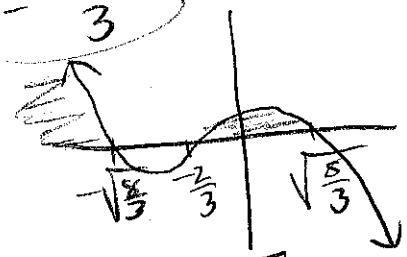
$-7(9x^3 + 6x^2 - 24x - 16) \geq 0$

-8	$-24x$	-16
$3x^2$	$9x^3$	$6x^2$
	$3x$	2

$-7(3x^2 - 8)(3x + 2) \geq 0$

$x = \pm\sqrt{\frac{8}{3}}$, $x = -\frac{2}{3}$

$\pm\frac{2\sqrt{6}}{3}$



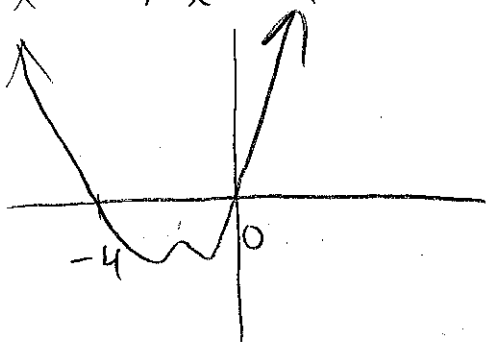
$D_F: (-\infty, -\sqrt{\frac{8}{3}}] \cup [-\frac{2}{3}, \sqrt{\frac{8}{3}}]$

15) $f(x) = \sqrt{4x^4 + 256x}$

$4x(x^3 + 64) \geq 0$

$4x(x + 4)(x^2 - 4x + 16) \geq 0$

$x = 0, x = -4$



$D_F: (-\infty, -4] \cup [0, \infty)$

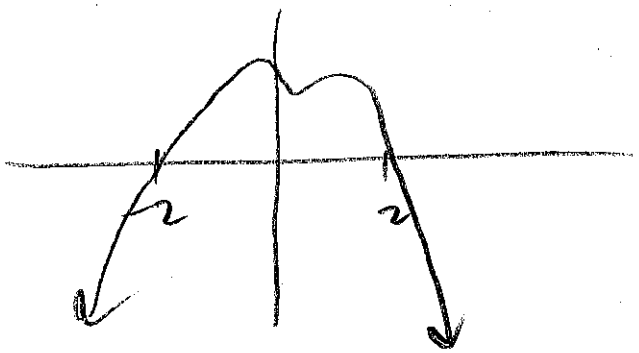
16) $f(x) = \sqrt{-7x^4 + 112}$

$-7(x^4 - 16) \geq 0$

$-7(x^2 - 4)(x^2 + 4) \geq 0$

$-7(x + 2)(x - 2)(x^2 + 4) \geq 0$

$x = -2, x = 2$



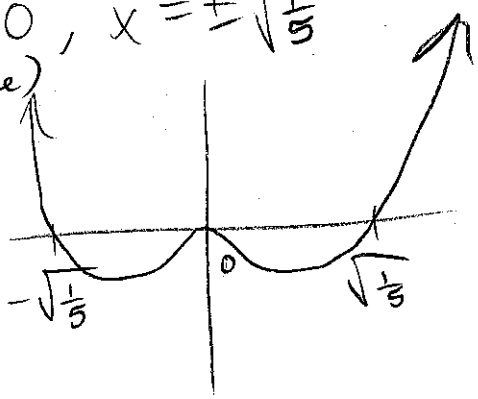
$D_F: [-2, 2]$

17) $f(x) = \sqrt{100x^6 - 4x^2}$

$4x^2(25x^4 - 1) \geq 0$

$4x^2(5x^2 - 1)(5x^2 + 1) \geq 0$

$x = 0, x = \pm\sqrt{\frac{1}{5}}$



$D_F: (-\infty, -\sqrt{\frac{1}{5}}] \cup [0] \cup [\sqrt{\frac{1}{5}}, \infty)$

Simplify. Rationalize & simplify for #'s 22 & 23. Show at least one step of work.

18) $(2-2i)-(1-8i)$

$1+6i$

19) $-5-(5+5i)-6$

$-5-5-5i-6$

$-16-5i$

20) $(2+6i)(-5-4i)$

$-10-8i-30i-24i^2$

$-10-38i-24(-1)$

$14-38i$

21) $(-8-2i)(-1+8i)$

$8-64i+2i-16i^2$

$8-62i-16(-1)$

$24-62i$

Simplify each. Show at least one step of work.

22) i^{21}

$=i^{20}\cdot i^1$

$=i$

23) i^{64} $=1$

64 is a multiple of 4.

24) i^{35}

$=i^{32}i^3$

$=i^3$

$=-i$