Name KEY

Around, around we go!

1. Find the radius of the circle if an arc of length 6 m on the circle is created by a central angle of $\frac{\pi}{6}$ rad.

$L = \theta r \quad \frac{6}{\pi} 6 = \frac{\pi}{6} \cdot r \cdot \frac{6}{\pi}$

$r = \frac{36}{\pi} m \approx 11.46 m$

2. How many revolutions will a car wheel of a diameter 30 in. make as the car travels a distance of one mile?

$C = d\pi$

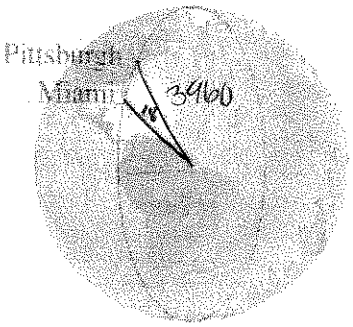
$C = 30\pi$ in

$1 mi = 5280 ft = 63{,}360 in$

$\frac{63{,}360}{30\pi} \approx 672.27$ revolutions

3. Pittsburgh, Pennsylvania, and Miami, Florida lie approximately on the same meridian. Pitt has latitude of 40.5º N and Miami, 22.5º N. Find the distance between these two cities. (The radius of the earth is 3960 mi.)

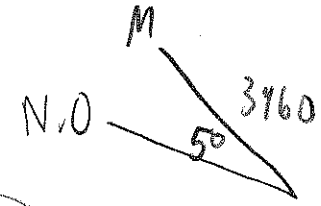
$40.5 - 22.5 = 18°$



$L = \frac{18}{360} \cdot 2\pi \cdot 3960$

$L = 396\pi$ mi ≈ 1244.1 mi

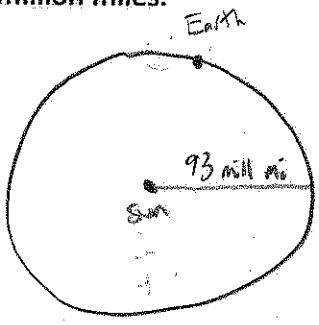
4. Memphis, Tennessee, and New Orleans, Louisiana, lie approximately on the same meridian. Memphis has latitude 30º N and New Orleans, 35º N. Find the distance between these two cities. (Earth's radius is 3960 mi.)



$L = \frac{5}{360} \cdot 2\pi \cdot 3960$

$L = 110\pi$ mi ≈ 345.58 mi

5. Find the distance that the earth travels in one day in its path around the sun. Assume that one year is 365 days and that the path of the earth around the sun is a circle (actually is elliptical) of radius 93 million miles.



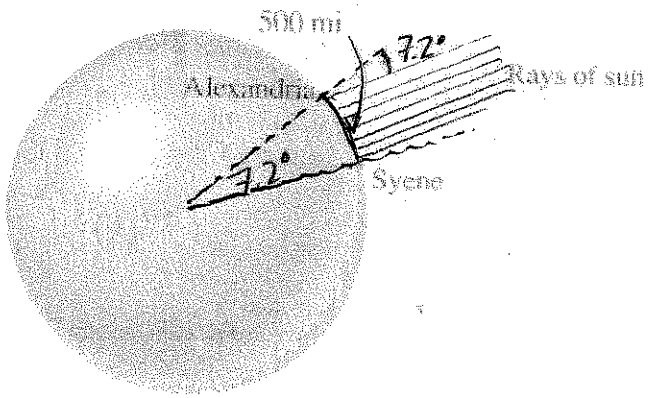
$\frac{1}{365} \cdot 2\pi \cdot 93{,}000{,}000 = 1{,}600{,}921.2 \text{ mi}$

6. The Greek mathematician Eratosthenes (ca. 276 – 195 B.C.) measured the circumference of the earth from the following observations. He noticed that on a certain day the sun shone directly down a deep well in Syene. At the same time in Alexandria, 500 miles north (on the same meridian), the rays of the sun shone at an angle of 7.2º to the zenith. Use this information and the figure to find the radius and circumference of the earth.

$L = \frac{\theta}{360} \cdot 2\pi r$

$500 \text{mi} = \frac{7.2}{360} \cdot 2\pi r$

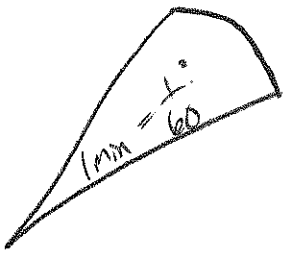
$r = 3978.87 \text{ mi.}$



$C = 2\pi \cdot 3978.87$

$= 25{,}000 \text{ mi.}$

7. Find the distance along an arc on the surface of the earth that is created by a central angle of 1 minute (1 min. = $\frac{1}{60}$ degree). This distance is called a nautical mile. Use the radius of the earth of 3960 mi.

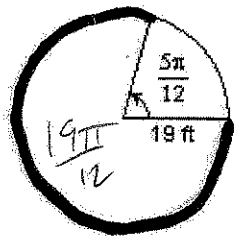


$\frac{\left(\frac{1}{60}\right)}{360} \cdot 2\pi \cdot 3960 = 1.15 \text{ mi}$

1 nautical mi = 1.15 mi

Find the area of each sector.

8)

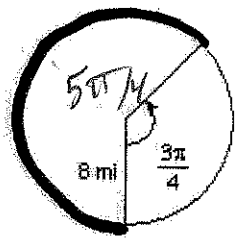


$\frac{24\pi}{12} - \frac{5\pi}{12} = \frac{19\pi}{12}$

$A = \frac{1}{2} \cdot \frac{19\pi}{12} \cdot 19^2$

$A \approx 897.84 \text{ ft}^2$

9)

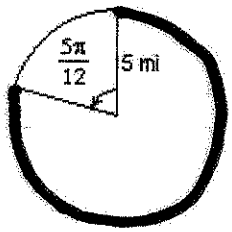


$\frac{8\pi}{4} - \frac{3\pi}{4} = \frac{5\pi}{4}$

$A = \frac{1}{2} \cdot \frac{5\pi}{4} \cdot 8^2$

$A \approx 125.66 \text{ mi}^2$

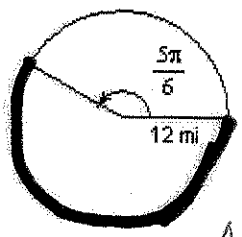
10)



$A = \frac{1}{2} \cdot \frac{19\pi}{12} \cdot 5^2$

$A \approx 62.18 \text{ mi}^2$

11)



$\frac{12\pi}{6} - \frac{5\pi}{6} = \frac{7\pi}{6}$

$A = \frac{1}{2} \cdot \frac{7\pi}{6} \cdot 12^2$

$A \approx 263.89 \text{ mi}^2$

12. Find the area of a sector with central angle 1 rad in a circle of radius 10 m.

$A = \frac{1}{2} \cdot 1 \cdot 10^2 = 50 \text{ m}^2$

13. A sector of a circle has a central angle of 60º. Find the area of a sector if the radius of the circle is 3 mi.

$A = \frac{60}{360} \cdot \pi \cdot 3^2 = \frac{3\pi}{2} \text{ mi}^2 \approx 4.71 \text{ mi}^2$

14. The area of a sector of a circle with central angle of 2 rad is 16 m^2. Find the radius of the circle.

$16 \text{ m}^2 = \frac{1}{2} \cdot 2 \cdot r^2$

$r = 4 \text{ m}$

15. A sector of a circle of radius 24 mi has an area of 288 mi^2. Find the central angle of the sector in radians.

$A = \frac{1}{2}\theta r^2 \qquad 288 = \frac{1}{2}\cdot\theta\cdot 24^2$

$\theta = 1$ rad

16. The area of a circle is 72 cm^2. Find the area of a sector of this circle that has a central angle of $\frac{\pi}{6}$ rad.

$A = \pi r^2$

$72 = \pi\cdot r^2$

$r = \sqrt{\frac{72}{\pi}} =$

$A = \frac{1}{2}\cdot\frac{\pi}{6}\cdot\left(\sqrt{\frac{72}{\pi}}\right)^2 = \frac{\pi}{12}\cdot\frac{72}{\pi}$

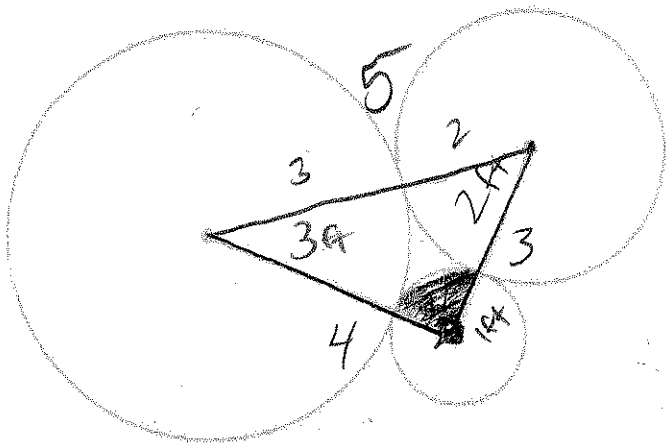
$= \frac{72}{12} = 6\text{ cm}^2$

17. Three circles with radii 1, 2, 3 ft are externally tangent to one another, as shown in the figure below. Find the area of the sector of the circle of radius 1 that is cut off by the line segments joining the center of that circle to the centers of the other two circles.

$A = \frac{1}{2}\theta r^2$

$A = \frac{1}{2}\cdot\frac{\pi}{2}\cdot 1^2$

$A = \frac{\pi}{4}\text{ ft}^2$

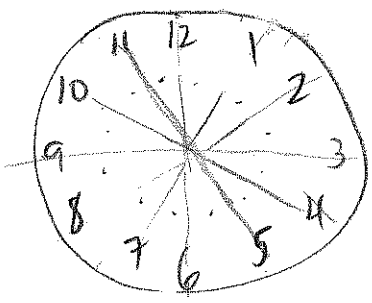


$3^2 + 4^2 = 5^2$

18. In one hour, the minute hand on a clock moves through a complete circle, and the hour hand moves through $\frac{1}{12}$ of a circle. Through how many radians do the minute and hour hand move between 1 p.m. and 6:45 p.m. (on the same day)?

1pm − 6:45pm = 5hr 45 min

= 5.75 hr = 345 min



$\frac{5}{12}\cdot 2\pi = \frac{10\pi}{12} = \frac{5\pi}{6}$ for hr

$+\frac{3}{4}\frac{\pi}{6} = \frac{23\pi}{24}$ for hr

$\frac{345}{60}\cdot 2\pi = \frac{23\pi}{2}$

$\frac{23\pi}{2}$ Mi