HPC - A

Name KEY

Max & Min Values

The process of finding maximum or minimum values is called **optimization**. We are trying to do things like maximize the profit in a company, or minimize the costs, or find the least amount of material to make a particular object. These are very important in the world of industry.

Example 1

What is the minimum (vertex) value of the function $f(x) = 2x^2 - 8x - 5$?

x of vertex $= \frac{-b}{2a} = \frac{8}{2(2)} = \boxed{2}$

$f(2) = 2(2)^2 - 8(2) - 5$

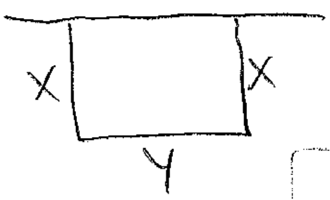
$f(2) = -13$

$\boxed{\text{Vertex (minimum)} = (2, -13)}$

Example 2

With 100 feet of fence, what are the $\boxed{\text{dimensions}}$ (x & y) of the pen with the $\boxed{\text{largest area}}$ a farmer can build if his barn will provide one side of the pen?

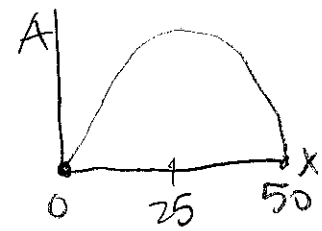
Maximum (vertex related)



$A = x \cdot y$

$100 = 2x + y \rightarrow y = 100 - 2x$

$\boxed{A = x(100 - 2x)} \rightarrow$



x int: 0, 50

x of vertex (one dimension) $= \boxed{25\text{ ft}}$ $y = 100 - 2(25) = \boxed{50\text{ ft}}$

Max area $\boxed{25\text{ ft by } 50\text{ ft}}$

Example 3

Last year the yearbook at Central High cost \$75 and only 500 were sold. A student survey found that for every \$5 reduction in price, 100 more students will buy yearbooks. What price should be charged to maximize the revenue from yearbook sales? (Hint: $Total\ Revenue = (cost\ of\ books)(number\ sold)$)

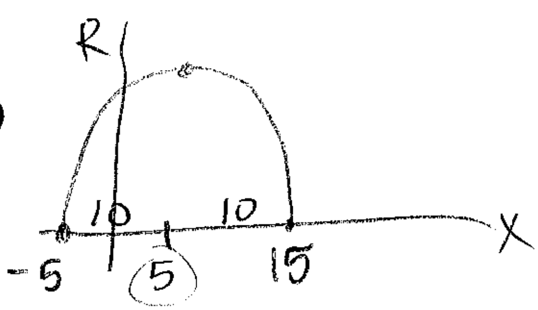
x represents the # of reductions.

$R = (75 - 5x)(500 + 100x)$

x-int: 15, −5

x = 5 means we need to reduce the price 5 times. $\rightarrow$ Cost $= 75 - 5(5)$

$\boxed{\text{Cost} = \$50 \text{ each}}$



1) The daily profit, P, of an oil refinery is given by $P = 8x - 0.02x^2$, where x is the number of barrels of oil refined. How many barrels will give maximum profit and what is the maximum profit?

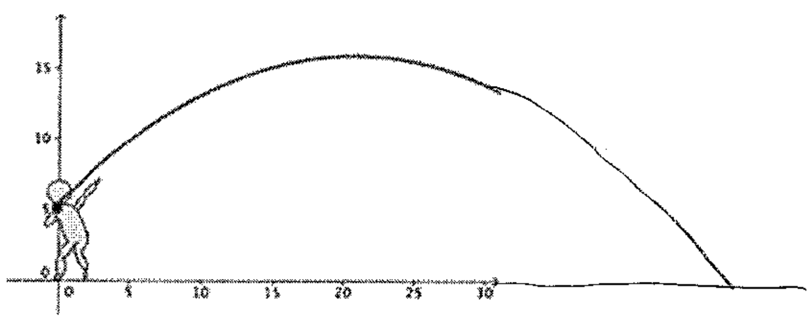
X of vertex $= \dfrac{-8}{2(-0.02)} = \boxed{200 \text{ barrels}}$

# of barrels to sell to maximize profit

$P = 8(200) - 0.02(200)^2$

$\boxed{P = \$800}$ Selling 200 barrels will result in \$800 profit.

2) A shot-put throw can be modeled using the equation $y = -0.0241x^2 + x + 5.5$, where x is the distance traveled (in feet) and y is the height (also in feet). Does this appear to be a quadratic that you would complete the square on? Think about other, more efficient ways when the coefficients are not "friendly."



a) How long was the throw? (where $y = 0$)

$X = \dfrac{-1 \pm \sqrt{(1)^2 - 4(-0.0241)(5.5)}}{2(-0.0241)} = \dfrac{-1 \pm 1.23701}{-0.0482}$

$X = -4.917$ (doesn't make sense for our situation)

* $\boxed{X = 46.41 \text{ ft}}$

b) How high was the shot-put when it was initially thrown?

when $x = 0$

$\boxed{5.5 \text{ ft}}$ y-int or "c" of standard form.

c) What is the highest the shot-put traveled off the ground? (y of vertex)

X of vertex $= \dfrac{-1}{2(-0.0241)} = 20.747$ ft Store in calc to make this easier!

distance traveled along the ground when shot-put was highest

$y = -0.0241(20.747)^2 + 20.747 + 5.5$

* $\boxed{y = 15.87 \text{ ft high}}$

For #'s 3-5, use "Guidelines for Applied functions" in order to create a quadratic equation and solve.

3) A rectangular storage area is to be constructed along the side of a tall building. A security fence is required along the remaining 3 sides of the area. What is the maximum area that can be enclosed with 800 m of fencing?

$A = xy$

$800 = 2x + y \rightarrow y = 800 - 2x$

$A = x(800 - 2x)$

x-int: $0, 400$

X of vertex = 200 ft

one dimension to give max area

$A_{max} = 200(800 - 2(200)) = \boxed{80{,}000 \text{ m}^2}$

4) A tomato grower needs to ship early when prices are high and spoilage is low. She now has 25 tons on hand and can add two tons a week by waiting. The current revenue is \$250 per ton but it will reduce by \$15 per ton for each week she delays. When should she ship to receive maximum revenue?
(Hint: *Total Revenue* = (*cost of* ~~*books*~~ ton of tomato)(*number sold*))

X represents # of weeks delayed.

$R = (250 - 15x)(25 + 2x)$

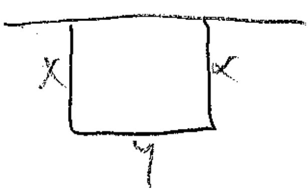
$R = -30x^2 + 125x + 6250$

X of vertex = $\frac{-125}{2(-30)} = \boxed{2.083 \text{ weeks}}$

# of weeks to delay to max revenue

For fun:
2.083 weeks is 2 weeks & 14 hours

5) Farmer Ted also has 2400 ft. of fencing and wants to fence off a rectangular field that borders a straight river, as show in the figure. What dimensions of fence will maximize the area?



$A = xy$

$2400 = 2x + y \rightarrow y = 2400 - 2x$

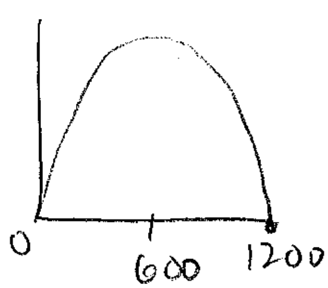
$A = x(2400 - 2x)$

x-int: $0, 1200$

x of vertex = $\boxed{600 \text{ ft}}$

(one dimension to give max area)

$y = 2400 - 2(600) = 1200 \text{ ft}$



600 ft by 1200 ft to maximize area