

## Graphing &amp; Finding Domain for High Powered Polynomials

Find the domain for each radical. Start by factoring the polynomial, graphing the polynomial, and finding positive intervals greater than or equal to zero.

1)  $f(x) = \sqrt{27x^2 - 3}$

$$27x^2 - 3 \geq 0$$

$$3(9x^2 - 1) \geq 0$$

$$3(3x-1)(3x+1) \geq 0$$

$$x = -\frac{1}{3}, x = \frac{1}{3}$$



$$D: (-\infty, -\frac{1}{3}] \cup [\frac{1}{3}, \infty)$$

2)  $f(x) = \sqrt{-27x^2 - 36x - 12}$

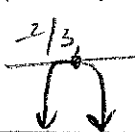
$$-27x^2 - 36x - 12 \geq 0$$

$$-3(9x^2 + 12x + 4) \geq 0$$

$$-3(3x+2)(3x+2) \geq 0$$

$$-3(3x+2)^2 \geq 0$$

$$x = -\frac{2}{3} \text{ (bounce)}$$



$$D: [-\frac{2}{3}]$$

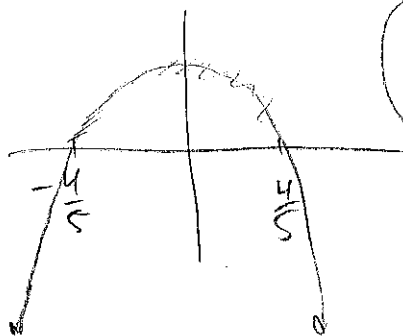
3)  $f(x) = \sqrt{-25x^2 + 16}$

$$-25x^2 + 16 \geq 0$$

$$-1(25x^2 - 16) \geq 0$$

$$-1(5x-4)(5x+4) \geq 0$$

$$x = -\frac{4}{5}, x = \frac{4}{5}$$



$$D: [-\frac{4}{5}, \frac{4}{5}]$$

4)  $f(x) = \sqrt{12x^2 + 36x + 27}$

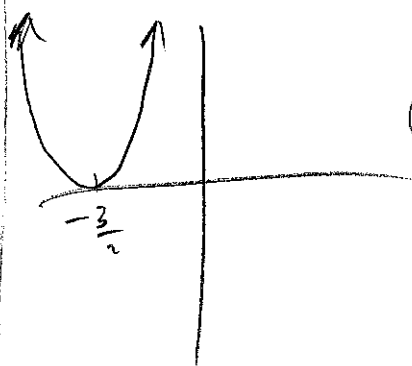
$$12x^2 + 36x + 27 \geq 0$$

$$3(4x^2 + 12x + 9) \geq 0$$

$$3(2x+3)(2x+3) \geq 0$$

$$3(2x+3)^2 \geq 0$$

$$x = -\frac{3}{2} \text{ (bounce)}$$



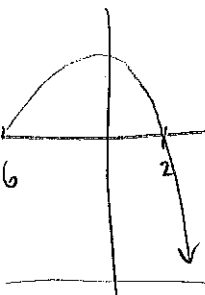
$$D: (-\infty, \infty)$$

$$5) f(x) = \sqrt{-2x^2 - 8x + 24}$$

$$-2(x^2 + 4x - 12) \geq 0$$

$$-2(x+6)(x-2) \geq 0$$

$$x = -6, x = 2$$



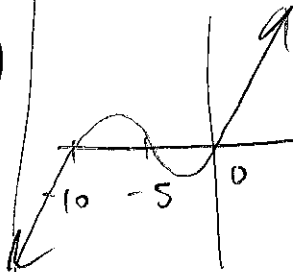
$$D: [-6, 2]$$

$$6) f(x) = \sqrt{2x^3 + 30x^2 + 100x}$$

$$2x(x^2 + 15x + 50) \geq 0$$

$$2x(x+5)(x+10) \geq 0$$

$$x = 0, x = -5, x = -10$$



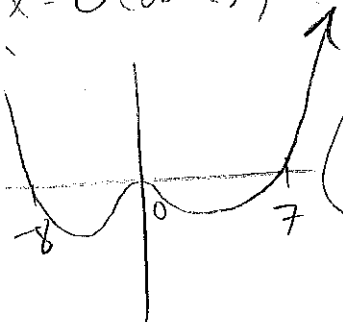
$$D: [-10, -5] \cup [0, \infty)$$

$$7) f(x) = \sqrt{x^4 + x^3 - 56x^2}$$

$$x^2(x^2 + x - 56) \geq 0$$

$$x^2(x+8)(x-7) \geq 0$$

$$x = 0 \text{ (bounce)}, x = -8, x = 7$$



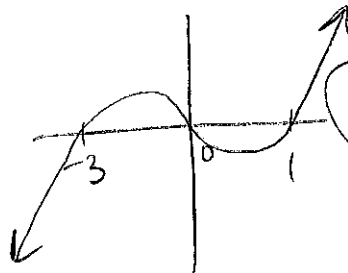
$$D: (-\infty, -8] \cup [0, 7] \cup [7, \infty)$$

$$8) f(x) = \sqrt{x^3 + 2x^2 - 3x}$$

$$x(x^2 + 2x - 3) \geq 0$$

$$x(x+3)(x-1) \geq 0$$

$$x = 0, x = -3, x = 1$$



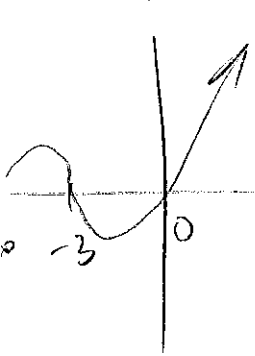
$$D: [-3, 0] \cup [1, \infty)$$

$$9) f(x) = \sqrt{x^3 + 9x^2 + 18x}$$

$$x(x^2 + 9x + 18) \geq 0$$

$$x(x+6)(x+3) \geq 0$$

$$x = 0, x = -6, x = -3$$



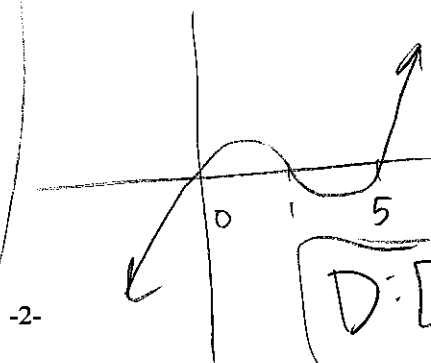
$$D: [-6, -3] \cup [0, \infty)$$

$$10) f(x) = \sqrt{x^3 - 6x^2 + 5x}$$

$$x(x^2 - 6x + 5) \geq 0$$

$$x(x-5)(x-1) \geq 0$$

$$x = 0, x = 5, x = 1$$



$$D: [0, 1] \cup [5, \infty)$$

$$11) f(x) = \sqrt{7x^3 - 6x^2 - 7x + 6}$$

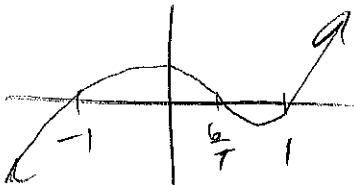
$$7x^3 - 6x^2 - 7x + 6 \geq 0$$

-6	-6x <sup>2</sup>	6
7x	7x <sup>3</sup>	-7x
	x <sup>2</sup>	-1

$$(7x-6)(x^2-1) \geq 0$$

$$(7x-6)(x+1)(x-1) \geq 0$$

$$x = \frac{6}{7}, x = -1, x = 1$$



$$D: [-1, \frac{6}{7}] \cup [1, \infty)$$

$$13) f(x) = \sqrt{x^4 - x^3 - 3x^2 + x + 2}; f(2) = 0$$

$$x^4 - x^3 - 3x^2 + x + 2 \geq 0$$

	x <sup>3</sup>	x <sup>2</sup>	-x	-1
x-2	x <sup>4</sup>	-x <sup>3</sup>	-3x <sup>2</sup>	+x+2
	-(x <sup>3</sup> -2x <sup>3</sup> )			
	x <sup>3</sup>	-3x <sup>2</sup>		
	-(x <sup>3</sup> -2x <sup>2</sup> )			
		-x <sup>2</sup>	+x	
		-(-x <sup>2</sup> +2x)		
			-1x+2	
			-(-1x+2)	
				0

$$(x-2)(x^3+x^2-x-1) \geq 0$$

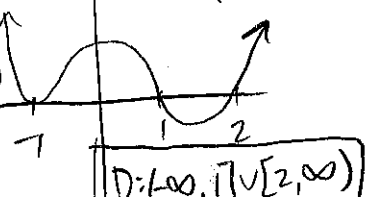
	x <sup>2</sup>	-1
x	x <sup>3</sup>	-x
	x <sup>2</sup>	-1

$$(x-2)(x+1)(x^2-1) \geq 0$$

$$(x-2)(x+1)(x+1)(x-1) \geq 0$$

$$(x-2)(x+1)^2(x-1) \geq 0$$

$$x = 2, x = -1 \text{ (bounce)}, x = 1$$



$$D: (-\infty, -1] \cup [2, \infty)$$

$$12) f(x) = \sqrt{-x^4 - 3x^3 + 4x}; f(-2) = 0$$

$$-x^4 - 3x^3 + 4x \geq 0$$

$$-x(x^3 + 3x^2 - 4) \geq 0$$

	x <sup>2</sup>	+x	-2
x+2	x <sup>3</sup>	+3x <sup>2</sup>	+0x-4
	-(x <sup>3</sup> +2x <sup>2</sup> )		
		x <sup>2</sup>	+0x
		-(-x <sup>2</sup> +2x)	
			-2x-4
			-(-2x-4)
			0

$$-x(x+2)(x^2+x-2) \geq 0$$

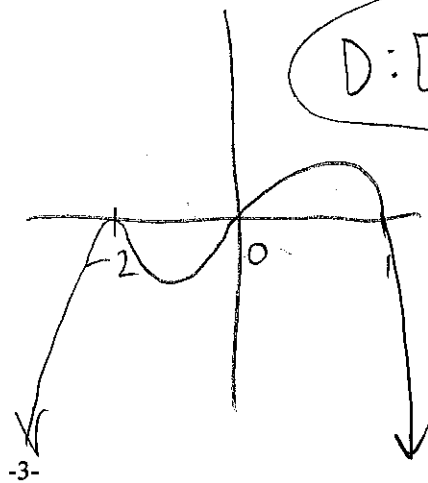
$$-x(x+2)(x+2)(x-1) \geq 0$$

$$-x(x+2)^2(x-1) \geq 0$$

$$x = 0, x = -2 \text{ (bounce)}, x = 1$$

$$\text{Deg } 4(-a)$$

$$D: [-2] \cup [0, 1]$$



$$14) f(x) = \sqrt{x^5 - 5x^4 - 8x^3 + 40x^2 + 7x - 35}; f(5) = 0$$

$$\begin{array}{r} x^4 - 8x^2 + 7 \\ x-5 \overline{) x^5 - 5x^4 - 8x^3 + 40x^2 + 7x - 35} \\ \underline{-(x^5 - 5x^4)} \phantom{- 8x^3 + 40x^2 + 7x - 35} \\ \phantom{0} - 8x^3 + 40x^2 \phantom{+ 7x - 35} \\ \phantom{0} \underline{-( - 8x^3 + 40x^2)} \phantom{+ 7x - 35} \\ \phantom{00} 7x - 35 \\ \phantom{00} \underline{-(7x - 35)} \\ \phantom{000} 0 \end{array}$$

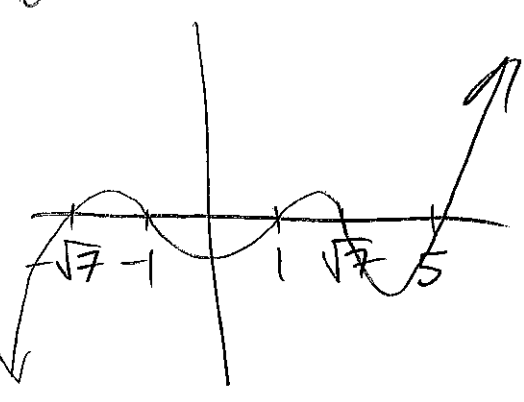
$$D: [-\sqrt{7}, -1] \cup [1, \sqrt{7}] \cup [5, \infty)$$

$$(x-5)(x^4 - 8x^2 + 7) \geq 0$$

$$(x-5)(x^2 - 1)(x^2 - 7) \geq 0$$

$$(x-5)(x+1)(x-1)(x^2 - 7) \geq 0$$

$$x=5, x=-1, x=1, x=\pm\sqrt{7}$$



$$15) f(x) = \sqrt{x^4 + 3x^3 + 64x + 192}; f(-3) = 0$$

Could use grouping strategy!

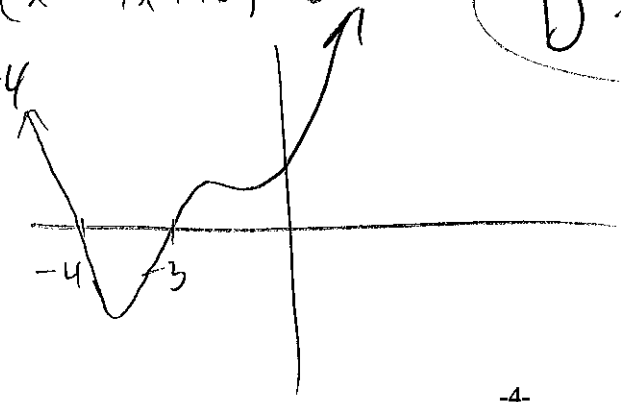
$$\begin{array}{r} x^3 + 64 \\ x+3 \overline{) x^4 + 3x^3 + 0x^2 + 64x + 192} \\ \underline{-(x^4 + 3x^3)} \phantom{+ 0x^2 + 64x + 192} \\ \phantom{0} 64x + 192 \\ \phantom{0} \underline{-(64x + 192)} \\ \phantom{00} 0 \end{array}$$

$$D: (-\infty, -4] \cup [-3, \infty)$$

$$(x+3)(x^3 + 64) \geq 0$$

$$(x+3)(x+4)(x^2 - 4x + 16) \geq 0$$

$$x = -3, x = -4$$



$$16) f(x) = \sqrt{-x^5 + 2x^4 + 8x^3 - 16x^2 - 16x + 32}; f(2) = 0$$

$$-(x^5 - 2x^4 - 8x^3 + 16x^2 + 16x - 32) \geq 0$$

$$\begin{array}{r}
 x^4 - 8x^2 + 16 \\
 \hline
 x-2 \overline{) x^5 - 2x^4 - 8x^3 + 16x^2 + 16x - 32} \\
 \underline{-(x^5 - 2x^4)} \phantom{+ 16x^2 + 16x - 32} \\
 \phantom{x-2 \overline{) }} 0 \phantom{+ 16x^2 + 16x - 32} \\
 \phantom{x-2 \overline{) }} \phantom{0} \phantom{+ 16x^2} -8x^3 + 16x^2 \\
 \phantom{x-2 \overline{) }} \phantom{0} \phantom{+ 16x^2} \underline{-(-8x^3 + 16x^2)} \\
 \phantom{x-2 \overline{) }} \phantom{0} \phantom{+ 16x^2} \phantom{0} \\
 \phantom{x-2 \overline{) }} \phantom{0} \phantom{+ 16x^2} \phantom{0} \phantom{+ 16x - 32} 16x - 32 \\
 \phantom{x-2 \overline{) }} \phantom{0} \phantom{+ 16x^2} \phantom{0} \phantom{+ 16x - 32} \underline{-(16x - 32)} \\
 \phantom{x-2 \overline{) }} \phantom{0} \phantom{+ 16x^2} \phantom{0} \phantom{+ 16x - 32} \phantom{0}
 \end{array}$$

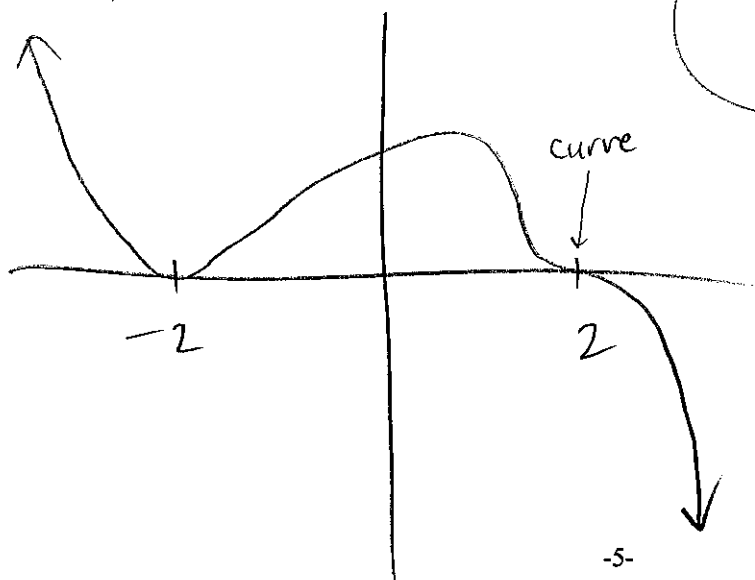
$$-(x-2)(x^4 - 8x^2 + 16) \geq 0$$

$$-(x-2)(x^2 - 4)(x^2 - 4) \geq 0$$

$$-(x-2)(x+2)(x-2)(x+2)(x-2) \geq 0$$

$$-(x-2)^3(x+2)^2 \geq 0$$

$$x = 2 \text{ (curve)}, x = -2 \text{ (bounce)}$$



$$D: (-\infty, 2]$$

