Thinking Rational

When graphing Rational Functions, we need to check for x & y-intercepts, vertical & horizontal (also *slant*, but those are for later) asymptotes, end behavior, and holes. The end behavior at times is a challenge when more than one vertical asymptote is involved, however, we will come up with a strategy to help us determine the appropriate direction for our graph!

Intercepts:

x-intercepts occur when $y = 0$.

When a Rational Function such as, $y = \frac{x-9}{x+5}$ is equal to zero ($0 = \frac{x-9}{x+5}$) we look to the numerator. The only way a Rational Function can output zero is if the numerator is equal to zero. Factor the numerator (if necessary) and find where the numerator is equal to zero. For the above example $0 = \frac{x-9}{x+5}$ when $x = 9$. Therefore; we have a $x - int\ at\ (9,0)$.

y-intercepts occur when $x = 0$.

When the same rational function $y = \frac{x-9}{x+5}$ has x-values of zero we get $y = \frac{0-9}{0+5}$ which gives us a $y = \frac{-9}{5}$. Therefore; we have a $y - int\ at\ \left(0, \frac{-9}{5}\right)$.

Asymptotes:

Vertical asymptotes occur when the denominator $= 0$.

For the above Rational Function, $y = \frac{x-9}{x+5}$, the denominator is equal to zero at $x = -5$. This means that there is a vertical asymptote, with equation $x = -5$. A vertical asymptote is a vertical line on the graph (usually dotted/dashed) that represents the set of x-values for which the graph is not defined. In this case, the graph is not defined when $x = -5$. The domain of a Rational Function is dependent on the location of the vertical asymptotes. Graphs will NEVER cross a vertical asymptote.

Horizontal asymptotes depend on the highest exponent in the numerator and denominator.

We use patterns when determining the horizontal asymptotes for Rational Functions. You will learn about limits in Calculus and how to find the horizontal asymptote this way. When the highest powers in the numerator and denominator are the same, the horizontal asymptote is a horizontal line through the fraction created by dividing the leading coefficients. If the highest power in the numerator is less than the highest power than the denominator, then the horizontal asymptote will by $y = 0$. Graphs will cross a horizontal asymptote and frequently do in comparison to vertical asymptotes.

Slant asymptotes occur when the highest power in the numerator is larger than the highest power in the denominator.

We calculate slant asymptotes by using long division. The quotient to this long division problem will provide an equation with a remainder. The remainder is not graphed. If there is not remainder, then the divisor was a factor of the dividend and there will be a *hole* in the resulting graph.

Holes:

Holes occur when a Rational Function is completely factored in the numerator and denominator and a common factor divides out.

We draw a hole on a graph for a Rational Function because that function is undefined at that specific value, allowing the graph to be on either side of the hole. This is different from an asymptote because of the end behavior. When factors divide and simplify for Rational Functions we must create a hole for the x-value that we removed.

Examples:

1. $y = \frac{x+7}{x-8}$

2. $y = \frac{x-2}{x^2-1}$

3. $y = \frac{x^2-3x-4}{2x^2+4x}$

For #'s 1-10, graph the Rational Functions. First find all relevant information needed (intercepts, asymptotes, end behavior, and holes).

1) $f(x) = \dfrac{x^2+x}{x^2-3x-4}$

2) $f(x) = \dfrac{-x^2+2x}{x^2+2x}$

3) $f(x) = \dfrac{3x^2 + 9x - 12}{x^2 + x - 6}$

4) $f(x) = \dfrac{x^2 - x - 12}{x^2 + 2x - 3}$

5) $f(x) = \dfrac{3x + 3}{x + 2}$

6) $f(x) = \dfrac{-2x - 8}{x + 1}$

7) $f(x)=\dfrac{-x^2+3x+4}{x^2+x-2}$

8) $f(x)=\dfrac{x^2+5x+4}{x^2-x-6}$

9) $f(x)=\dfrac{x+4}{2x^2+10x+8}$

10) $f(x)=\dfrac{3x^2-3}{x^2+x}$

Bonus:

Find the slant asymptote and sketch a graph (no horizontal asymptote).

$$f(x) = \frac{x^2 - 4x - 5}{x - 3}$$