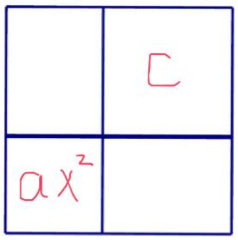
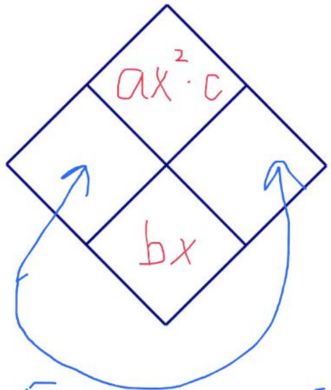
Factoring Quadratic Expressions

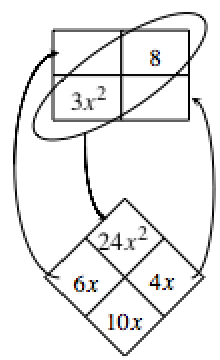
$y = ax^2 + bx + c$



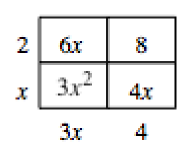


Factors of $ax^2 \cdot c$ that add up to bx.

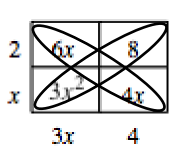
The process of factoring quadratics using this strategy is outlined below.
Factor $3x^2 + 10x + 8$.



1. Place the x^2 and constant terms of the quadratic expression in opposite corners of a generic rectangle. Determine the sum and product of the two remaining corners: The sum is simply the x-term of the quadratic expression, while the product is equal to the product of the x^2 and constant terms.
2. Place this sum and product into a Diamond Problem and solve it.
3. Place the solutions from the Diamond Problem into the generic rectangle and find the dimensions of the generic rectangle.
4. Write your answer as a product: $(3x + 4)(x + 2)$.



Notice that the product of the diagonal terms are equivalent (this is another way you can check your work!).



$$\mathbf{3x^2 \cdot 8 = 6x \cdot 4x}$$
$$= 24x^2$$

Survival Guide: Factoring

1. Look for GCF (greatest common factor) before doing any of the strategies below. This step may be performed last in some cases; however, the initial process may prove to be more difficult. It is recommended to factor out -1 if your "a" value is negative. This will make the resulting process much easier.

2. Determine the number of terms.

3. If there are ***__two terms__***, then you may have to use a factoring formula (difference of squares, difference or sum of cubes as shown below). *Ask yourself one very important question…*
   Are either terms perfect squares or perfect cubes (they could be both...think about that!)?

   When we have 2 terms that are perfect squares:
   $A^2 - B^2 = (A - B)(A + B)$ where A & B will always be positive values (sign taken care of in formula).

   When we have 2 terms that are perfect cubes:
   $A^3 - B^3 = (A - B)(A^2 + AB + B^2)$ The "quadratic" factor has imaginary solutions (does not factor).
   $A^3 + B^3 = (A + B)(A^2 - AB + B^2)$ The "quadratic" factor has imaginary solutions (does not factor).

   Sometimes with two terms the polynomial will just have a GCF and that will be all (example: $x^2 + 5x$).

4. If there are ***__three terms__***, then you can factor directly into two parentheses (factors of "c" that add up to "b") or create four terms and factor by grouping. If grouping, find factors of "ac," that add up to "b." Use the area or diamond method to find the "dimensions."

5. If there are ***__four terms__***, then look to factor by grouping first. If this is not possible, find one rational root (zero, x-intercept, etc.) and use long division to find another polynomial that should factor.

6. If there are ***__more than four terms__***, then use long division to create another polynomial that may factor. Sometimes more than one long division may need to occur to get to a polynomial that is factorable.