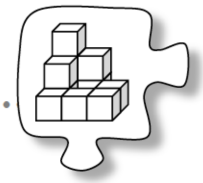
9.1.1 How can I build it?



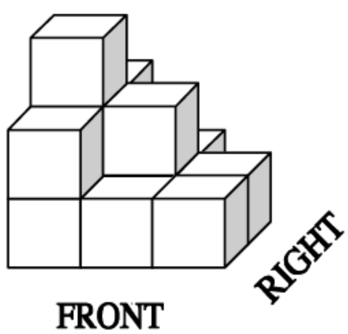
Three-Dimensional Solids

With your knowledge of polygons and circles, you are able to create and explore new, interesting shapes and make elaborate designs such as the one shown in the stained glass window at right. However, in the physical world, the objects you encounter every day are three-dimensional. In other words, physical objects cannot exist entirely on a flat surface, such as a tabletop.

To understand the shapes that you encounter daily, you will need to learn more about how three-dimensional shapes, called **solids**, can be created, described, and measured.

As you work with your team today, be especially careful to explain to your teammates how you "see" each solid. Remember that spatial visualization takes time and effort, so be patient with your teammates and help everyone understand how each solid is built.

9-1. Using blocks provided by your teacher or using the 3D Blocks (CPM) eTool, work with your team to build the three-dimensional solid below. Assume that blocks cannot hover in midair. That is, if a block is on the second level, assume that it has a block below it to prop it up.



a. Is there more than one arrangement of blocks that could look like the solid drawn above? Why or why not?

b. To avoid confusion, a **mat plan** can be used to show how the blocks are arranged in the solid. The number in each square represents the number of the blocks stacked in that location if you are looking from above. For example, in the right-hand corner, the solid is only 1 block tall, so there is a "1" in the corresponding corner of its mat plan.

Verify that the solid your team built matches the solid represented in the mat plan below.

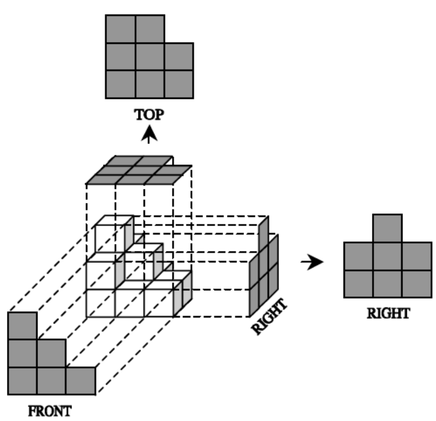
2	1	0	RIGHT
3	2	1	
2	1	1	

FRONT

Mat Plan

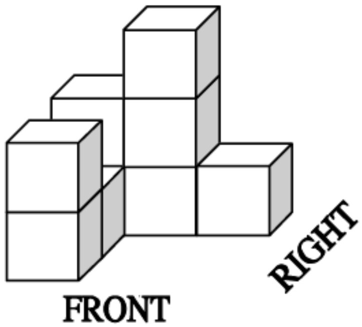
c. What is the **volume** of the solid? That is, if each block represents a cubic unit, how many blocks (cubic units) make up this solid?

9-2. Another way to represent a three-dimensional solid is by its **side** and **top views**.



For example, the solid from problem 9-1 can also be represented by a top, front, and right-hand view, as shown above. Each view shows *all* of the blocks that are visible when looking directly at the solid from that direction.

Examine the diagram of blocks below. On graph paper, draw the front, right, and top views of this solid. Assume that there are no hidden blocks. Explore using the 9-2 Student eTool (CPM).



9-3. For each of the mat plans below:

- Build the three-dimensional solid using the 3D Blocks (CPM) or with the blocks provided by your teacher.
- Find the volume of the solid in cubic units.
- Draw the front, right, and top views of the solid on a piece of graph paper.

a.

0	3	0
2	3	1
0	2	0

RIGHT

FRONT

b.

0	2	1
0	3	0
3	2	1

RIGHT

FRONT

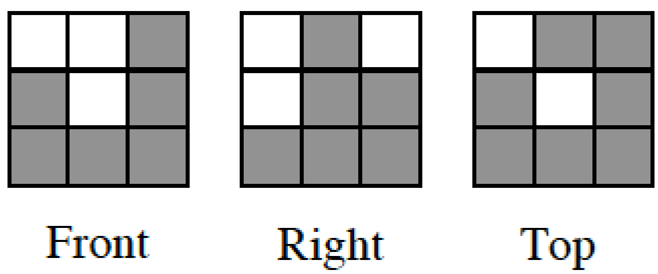
c.

1	1	3
2	1	2
0	0	1

RIGHT

FRONT

9-4. Meagan built a shape with blocks and then drew the views shown below.



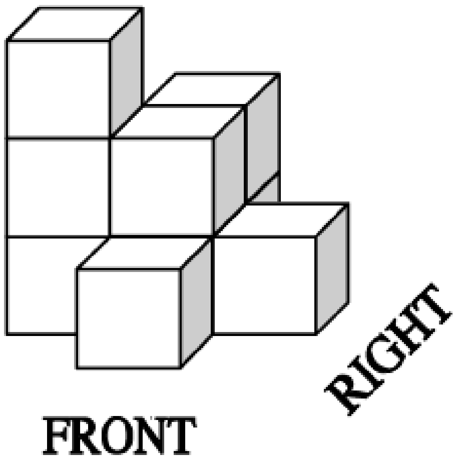
a. Build Meagan's shape using blocks provided by your teacher or the [9-4 Student eTool] (CPM). Use as few blocks as possible.

b. What is the volume of Meagan's shape?

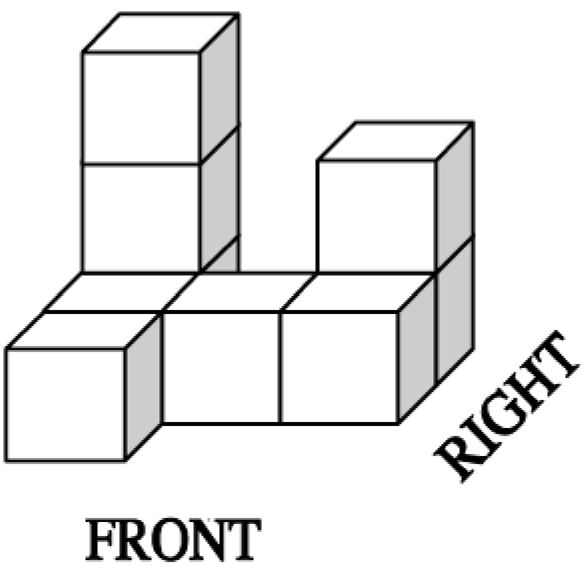
c. Draw a mat plan for her shape.

9-5. Draw a mat plan for each of the following solids. There may be more than one possible answer! Then find the possible volumes of each one. Explore using the 3D Blocks (CPM) eTool.

a.



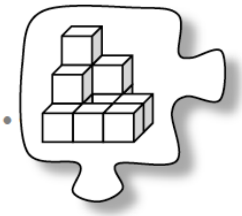
b.



9-6. LEARNING LOG

During this lesson, you have found the volume of several three-dimensional solids. However, what *is* volume? What does it measure? Write a Learning Log entry describing volume. Add at least one example. Title this entry "Volume of a Three-Dimensional Shape" and include today's date.

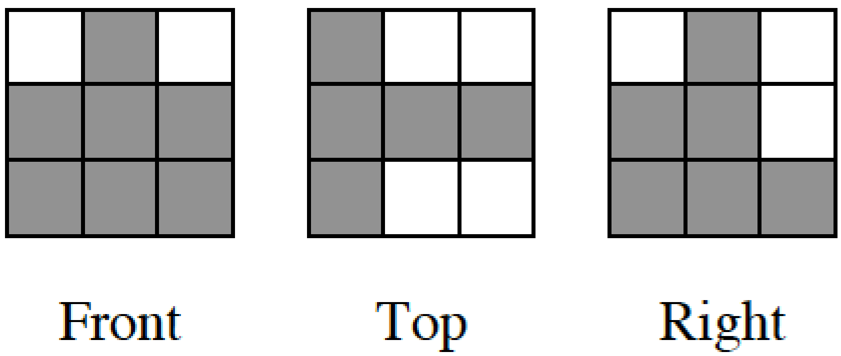
9.1.2 How can I measure it?



Volumes and Surface Areas of Prisms

Today you will continue to study three-dimensional solids and will practice representing a solid using a mat plan and its side and top views. You will also learn a new way to represent a three-dimensional object, called a net. As you work today, you will learn about a special set of solids called prisms and will study how to find the surface area and volume of a prism.

9-14. The front, top, and right-hand views of Heidi's solid are shown below.



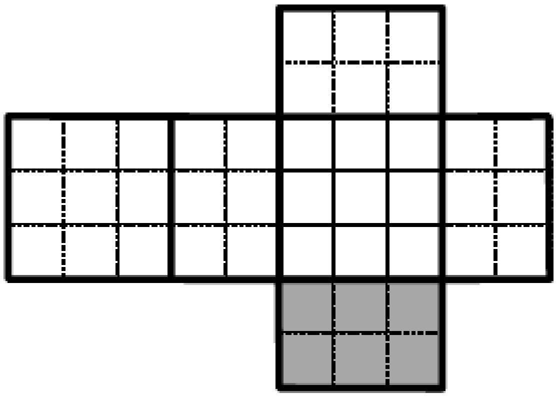
a. Build Heidi's solid using the [9-14 Student eTool] (CPM) or blocks provided by your teacher. Use the smallest number of blocks possible. What is the volume of her solid?

b. Draw a mat plan for Heidi's solid. Be sure to indicate where the front and right sides are located.

c. Oh no! Heidi accidentally dropped her entire solid into a bucket of paint! What is the **surface area** of her solid? That is, what is the area that is now covered in paint?

9-15. So far, you have studied three ways to represent a solid: a three-dimensional drawing, a mat plan, and its side and top views.

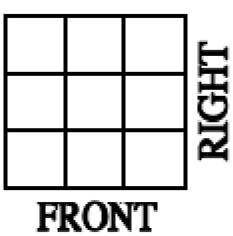
Another way to represent a three-dimensional solid is with a **net**, such as the one shown below. When folded, a net will form the three-dimensional solid it represents.



a. With your team, predict what the three-dimensional solid formed by this net will look like. Assume the shaded squares make up the base (or bottom) of the solid.

b. Obtain a [Lesson 9.1.2 Resource Page] and scissors from your teacher and cut out the net. Fold along the solid lines to create the three-dimensional solid. Did the result confirm your prediction from part (a)?

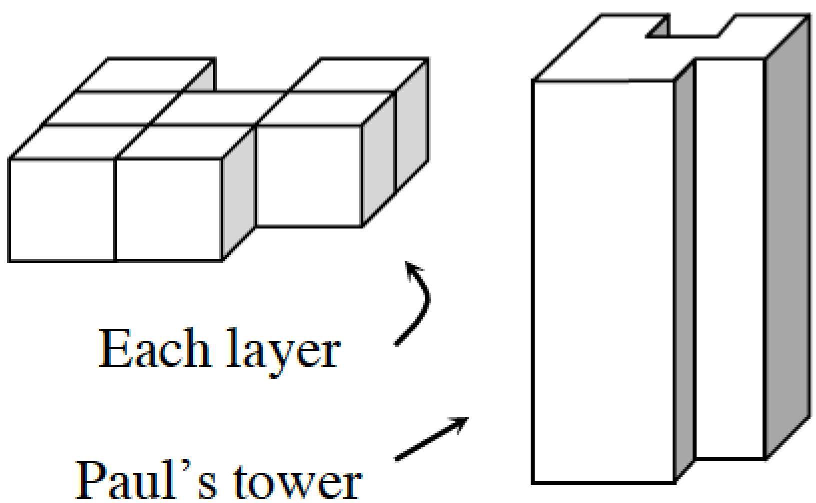
c. Now build the shape with blocks using the [3D Blocks] (CPM) eTool and complete the mat plan below for this solid.



d. What is the volume of this solid? How did you get your answer?

e. What is the surface area of the solid? How did you find your answer? Be prepared to share any shortcuts with the class.

9-16. Paul built a tower by stacking six identical layers of the shape below on top of each other.



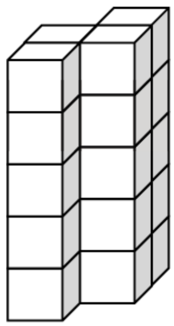
a. What is the volume of his tower? How can you tell without building the shape?

b. What is the surface area of his tower?

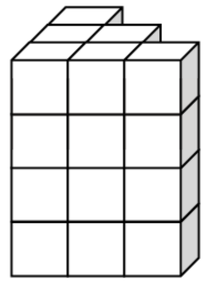
c. Paul’s tower is an example of a **prism** because it is a solid and two of its faces (called **bases**) are congruent and parallel. A prism must also have sides that connect the bases (called **lateral faces**). Each lateral face must be a parallelogram (and thus may also be a rectangle, rhombus or a square).

For each of the prisms below, find the volume and surface area.

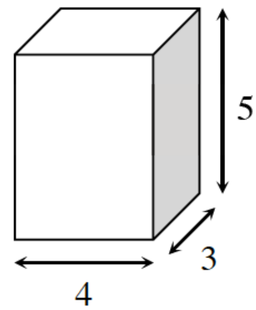
(1)



(2)



(3)



9-17. Heidi created several more solids, represented below. Find the volume of each one. Explore using the 3D Blocks (CPM) eTool.

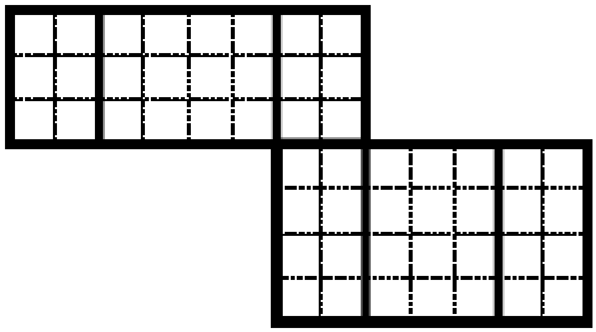
a.

0	3	5
22	10	25
18	15	8
16	12	0

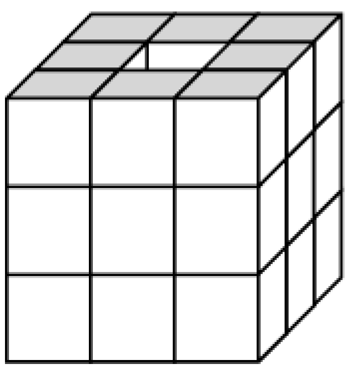
RIGHT

FRONT

b.



c.

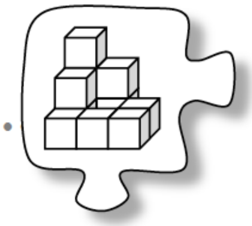


9-18. Pilar built a tower by stacking identical layers on top of each other. If her tower used a total of 312 blocks and if the bottom layer has 13 blocks, how tall is her tower? Explain how you know.

9-19. LEARNING LOG

What is the relationship between the area of the base of a prism, its height, and its volume? In a Learning Log entry, summarize how to find the volume of a solid. Be sure to include an example. Title this entry "Finding Volume" and include today's date.

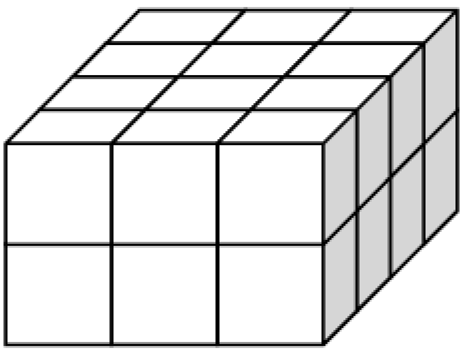
9.1.3 What if the bases are not rectangles?



Prisms and Cylinders

In Lessons 9.1.1 and 9.1.2, you investigated volume, surface area, and special three-dimensional solids called prisms. Today you will explore different ways to find the volume and surface area of a prism and a related solid called a cylinder. You will also consider what happens to the volume of a prism or cylinder if it slants to one side or if it is enlarged proportionally.

9-28. Examine the three-dimensional solid below.



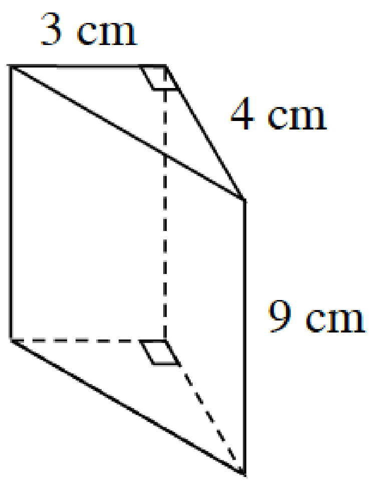
a. On graph paper, draw a net that, when folded, will create this solid.

b. Compare your net with those of your teammates. Is there more than one possible net? Why or why not?

c. Find the surface area and volume of this solid.

9-29. SPECIAL PRISMS

The prism in problem 9-28 is an example of a **rectangular prism**, because its bases are rectangular. Similarly, the prism below is called a **triangular prism** because the two congruent bases are triangular.

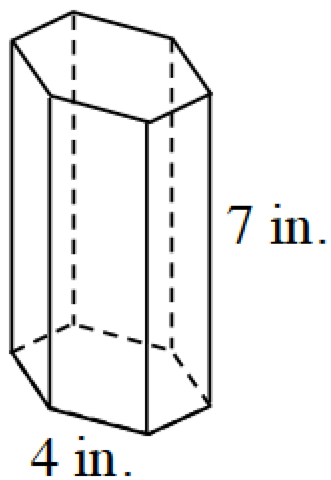


a. Carefully draw the prism at right onto your paper. One way to do this is to draw the two triangular bases first and then to connect the corresponding vertices of the bases. Notice that hidden edges are represented with dashed lines.

b. Find the surface area of the triangular prism. Remember that the surface area includes the areas of *all* surfaces – the sides and the bases. Carefully organize your work and verify your solution with your teammates.

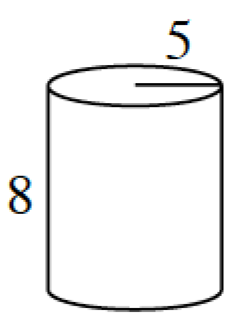
c. Find the volume of the triangular prism. Be prepared to share your team's method with the class.

d. Does your method for finding surface area and volume work on other prisms? For example, what if the bases are hexagonal, like the one shown below? Work with your team to find the surface area and volume of this hexagonal prism. Assume that the bases are regular hexagons with side length 4 inches.



9-30. CYLINDERS

Carter wonders, "*What if the bases are circular?*" Copy the **cylinder** below onto your paper. Discuss with your team how to find its surface area and volume if the radius of the base is 5 units and the height of the cylinder is 8 units.



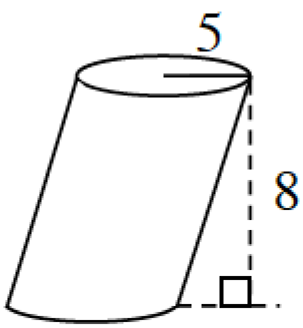
9-31. CAVALIERI'S PRINCIPLE

Bonaventura Cavalieri (1598-1647) was a mathematician who helped to develop calculus, but is best remembered today for a principle named for him. Cavalieri's Principle can be thought of as a way of finding volumes in a relatively easy way.

a. Suppose you have a stack of 25 pennies piled one on top of the other. You decide to slant the stack by sliding some of the pennies over. Does the volume of the 25 pennies change even though they are no longer stacked one on top of another?

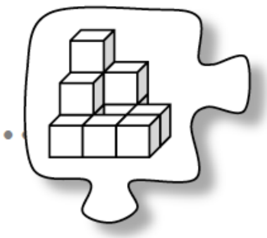
b. Would the same thing be true of a stack of 15 books that you slide to the side or twist some of them? What about a stack of 1000 sheets of paper?

c. The idea of viewing solids as slices that can be moved around without affecting the volume is called **Cavalieri's Principle**. Use this principle to find the volume of the cylinder below. Note that when the lateral faces of a prism or cylinder are not perpendicular to its base, the solid is referred to as an **oblique**cylinder or prism. How is the volume of this prism related to the one in problem 9-30?



9-32. Hernando needs to replace the hot water tank at his house. He estimates that his family needs a tank that can hold at least 75 gallons of water. His local water tank supplier has a cylindrical model that has a diameter of 2 feet and a height of 3 feet. If 1 gallon of water is approximately 0.1337 cubic feet, determine if the supplier's tank will provide enough water.

9.1.4 How does the volume change?



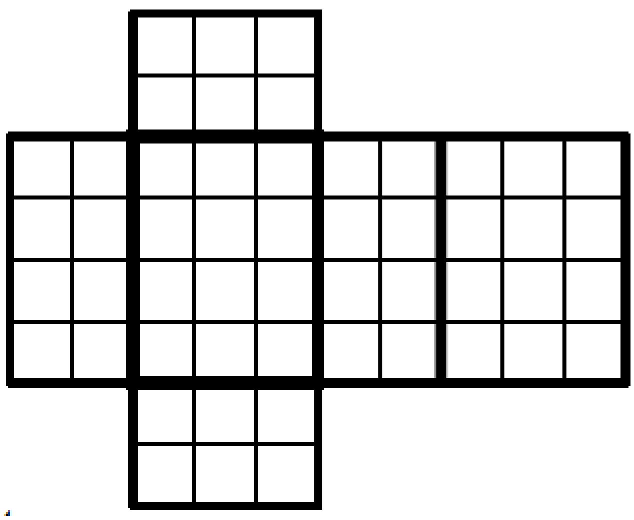
Volumes of Similar Solids

As you continue your study of three-dimensional solids, you will explore how the volume of a solid changes as the solid is enlarged proportionally.

9-41. HOW DOES THE VOLUME CHANGE?

In Lesson 9.1.3, you began a study of the surface area and volume of solids. Today, you will continue that investigation in order to generalize about the ratios of similar solids.

a. Describe the solid formed by the net below. What are its dimensions (length, width, and height)?



b. Have each team member select a different enlargement ratio from the list below. On graph paper, carefully draw the net of a similar solid using your enlargement ratio. Then cut out your net and build the solid (so that the gridlines end up on the outside the solid) using scissors and tape.

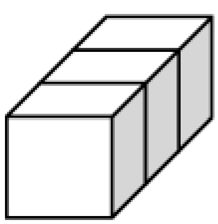
(1) 1 (2) 2 (3) 3 (4) 4

c. Find the volume of your solid and compare it to the volume of the original solid. What is the ratio of these volumes? Share the results with your teammates so that each person can complete a table like the one below.

Linear Scale Factor	Original Volume	New Volume	Ratio of Volumes
1			
2			
3			
4			
r			

d. How does the volume change when a three-dimensional solid is enlarged or reduced to create a similar solid? For example, if a solid's length, width, and depth are enlarged by a linear scale factor of 10, then how many times bigger does the volume get? What if the solid is enlarged by a linear scale factor of *r*? Explain.

9-42. Examine the $1 \times 1 \times 3$ solid below.



a. Build this solid with the 9-42 Student eTool (CPM) or with blocks provided by your teacher.

b. If this shape is enlarged by a linear scale factor of 2, how wide will the new shape be? How tall? How deep?

c. How many of the $1 \times 1 \times 3$ solids would you need to build the enlargement described in part (b) above? Use blocks to prove your answer.

d. What if the $1 \times 1 \times 3$ solid is enlarged with a linear scale factor of 3? How many times larger would the volume of the new solid be? Explain how you found your answer.

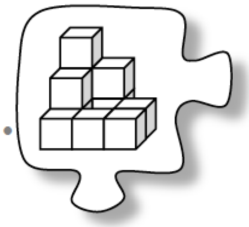
9-43. At the movies, Maurice counted the number of kernels of popcorn that filled his tub and found that it had 320 kernels. He decided that next time, he will get an enlarged tub that is similar, but has a linear scale factor of 1.5. How many kernels of popcorn should the enlarged tub hold?

9-44. LEARNING LOG

In your Learning Log, explain how the volume changes when a solid is enlarged proportionally. That is, if a three-dimensional object is enlarged by a linear scale factor of 2, by what factor does the volume increase? Title this entry “Volumes of Similar Solids” and include today’s date.

9.1.5 How does the volume change?

Ratios of Similarity



Today, work with your team to analyze the following problems. As you work, think about whether the problem involves volume or area. Also think carefully about how similar solids are related to each other.

9-53. A statue to honor Benjamin Franklin will be placed outside the entrance to the Liberty Bell exhibit hall in Philadelphia. The designers decide that a smaller, similar version will be placed on a table inside the building. The dimensions of the life-sized statue will be four times those of the smaller statue. Planners expect to need 1.5 pints of paint to coat the small statue. They also know that the small statue will weigh 14 pounds.

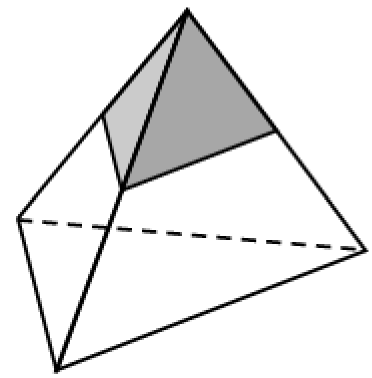
a. How much paint will be needed to paint the life-sized statue?

b. If the small statue is made of the same material as the enlarged statue, then its weight will change just as the volume changes as the statue is enlarged. How much will the life-size statue weigh?

9-54. The Blackbird Oil Company is considering the purchase of 20 new jumbo oil storage tanks. The standard model holds 12,000 gallons. Its dimensions are $\frac{4}{5}$ the size of the similarly shaped jumbo model, that is, the ratio of the dimensions is 4:5.How much more storage capacity would the twenty jumbo models give Blackbird Oil?

a. How much more storage capacity would the twenty jumbo models give Blackbird Oil?

b. If jumbo tanks cost 50% more than standard tanks, which tank is a better buy?



9-55. In problem 7-14 your class constructed a large tetrahedron like the one at right. Assume the dimensions of the shaded tetrahedron at right are half of the dimensions of the similar enlarged tetrahedron.

a. If the volume of the large tetrahedron is 138 in.^{3}, find the volume of the small shaded tetrahedron.

b. Each face of a tetrahedron is an equilateral triangle. If the small shaded tetrahedron has an edge length of 16 cm, find the total surface area for each of the tetrahedra.

c. Your class tried to construct a tetrahedron using four smaller congruent tetrahedra. However, the result left a gap in the center, as shown in the diagram below. If the volume of each small shaded tetrahedron is 50 in.^{3}, what is the volume of the gap? Explain how you know.